Conclusion: To cater for the time-varying nature of the flow rates at a road junction, a new fuzzy logic traffic controller is proposed. Its rules are adapted to the current traffic condition according to the regions defined on the flow-rate space. Not only that this controller shows notable improvement in terms of average-delay reduction, it also provides incentives for further investigations into on-line flow-rate prediction, definitions of regions in flow-rate space and recovery from unexpected traffic disturbances.

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Electronics Letters Online No. 19961052
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References

Robust sliding mode controller with fuzzy tuning
Q.P. Ha

Indexing term: Fuzzy control

A new technique is presented for linearised systems with uncertainties using sliding mode control with fuzzy tuning. The validity of the proposed approach is verified by the control of a two-mass servo-system.

Introduction: In sliding mode control [1], one of the methods for reducing chattering without sacrificing robust performance is to adjust the control action during the reaching phase using fuzzy logic [2]. This Letter presents a new technique for linearised systems which suffer from uncertainties using sliding mode control combined with fuzzy tuning to compensate for the influence of unmodelled dynamics and chattering.

Problem formulation: Consider the linearised model of a dynamic system:

\[ \dot{x} = (A + \Delta A)x + (b + \Delta b)u \]  

where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R} \) is the control signal, \( A \in \mathbb{R}^{n \times n} \) and \( b \in \mathbb{R}^n \) are the nominal system matrices, and \( \Delta A \) and \( \Delta b \) are uncertainties. The following assumptions are made:

(i) The uncertainties \( \Delta A = \Delta A(p) \) and \( \Delta b = \Delta b(p) \) are continuous matrix functions of the vector of varying system parameters, \( p \in \mathbb{R}^p \), where \( \mathbb{R}^p \) is a compact set.

(ii) There exist a vector \( \kappa(p) \in \mathbb{R}^n \) and a scalar \( h(p) \) such that the following matching conditions are satisfied:

\[ \Delta A = b\kappa'(p) \quad \text{and} \quad \max[|\kappa(p)|] \leq \rho \quad \forall p \in \mathbb{R}^p \]

\[ \Delta b = bh(p) \quad \text{and} \quad |h(p)| \leq \sigma < 1 \quad \forall p \in \mathbb{R}^p \]  

The objective is to design a controller that provides robust performance in the presence of uncertainties, given their bounds \( \rho \) and \( \sigma \). The control law is proposed as:

\[ u = u_{eq} + u_c + u_{ff} = -k_x^T x + u_{ff} \]  

where \( u_{eq}, u_c, \) and \( u_{ff} \) are the control signals for the equivalent motion, reaching phase with respect to uncertainties, and fuzzy tuning to improve the control performance, respectively.

Controller design: Define a switching hyperplane variable

\[ S(x) = c^T x = \sum_{i=1}^{n} c_i x_i \quad (c_n = 1) \]  

A necessary condition for the state trajectory to stay in the sliding mode \( S(x) = 0 \) is \( S(x) = 0 \) or

Fig. 2 Results of simulation where flow-rate profiles are different for the two traffic streams, but their sum (i.e. total demand at the junction) is constant.

Fig. 3 Results of simulation where the flow-rate profiles are identical for the two traffic streams, hence the total demand is time-varying.

The results of two simulation runs are given in Figs. 2 and 3, respectively, indicating a significant reduction in average delays in general. The means of the average delays over the simulated period (7200s) confirm the same conclusion in Table 2. It can also be shown that when the proposed controller is employed, the delay is less sensitive to flow-rate variation. In other words, by adapting the control rules to the traffic conditions, the impact of unexpected traffic disturbance on the road-users can be alleviated.

Table 2: Means of average delays (s/veh) in simulation runs

<table>
<thead>
<tr>
<th></th>
<th>Pappis controller</th>
<th>Proposed controller</th>
</tr>
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<tbody>
<tr>
<td>Case I</td>
<td>28.36</td>
<td>21.64</td>
</tr>
<tr>
<td>Case II</td>
<td>31.32</td>
<td>22.03</td>
</tr>
</tbody>
</table>
\[ e^T \hat{x} = e^T (A \hat{x} + bu_{eq}) = e^T A \hat{x} + e^T b \hat{u}_T \hat{x} = 0 \]  
where the existence of the inverse of \( e^T b \) is a necessary condition. The equivalent feedback gain \( \hat{k}_T \) can be determined by a pole placement technique to assign \((n-1)\) sliding eigenvalues \( \lambda_i \) (i = 1, 2, n-1), and an eigenvalue at the origin of the complex plane, \( \lambda_0 = 0 \), generated from the condition \( S(x) = 0 \), to the characteristic equation

\[ \left| \lambda - A + b \hat{k}_T \right| = 0 \]  
In order to meet the reaching condition, robust feedback with the switching gain \( \hat{k}_T \) is introduced.

**Theorem:** Consider the linearised system with uncertainties (eqn. 1) satisfying the first and second assumptions. If the robust sliding-mode controller is designed such that

\[ u = u_{eq} + u_r = -(\hat{k}_T + b \hat{u}_T) x \]  
where \( \hat{k}_T \) is the equivalent gain given in eqn. 6, and \( \hat{u}_T \) is the robust gain to deal with the system uncertainties, whose \( i \)-th element is given by

\[ k_{r,i} = \frac{\sigma + |k_{r,i}| \sigma - \text{sgn}(e^T b x_{r,i}, S)}{1 - \sigma} \quad (i = 1, 2, ..., n) \]  
then the state vector \( x(t) \) asymptotically converges to zero.

**Proof:** Define a positive definite Liapunov function \( V = 1/2 \dot{S} \). For the occurrence of sliding motion, the trajectories in the neighbourhood of \( S = 0 \) must be directed towards the mode of the limit \( \dot{V} \leq 0 \). From eqns. 1, 4, 5 and 8, we have

\[ \dot{S} = e^T \dot{x} = e^T [(A + \Delta A) b + (A + \Delta b)(-k_T x - b x)] = e^T b x_r - (1 + h[p])k_T x - h[p] k_T x \]  

Thus, the first order derivative of the Liapunov function can be expressed as

\[ \dot{V} = SS = e^T b x_r - (1 + h[p])k_T x - h[p] k_T x \]  

The condition \( \lim_{V \to 0} \dot{V} \leq 0 \) will hold if \( \{1 + h[p] k_T x - h[p] k_T x \} \) is \( \leq 0 \), \( \forall i = 1, 2, ..., n, \forall x \in \mathbb{P} \). The switching gain can then be derived as

\[ k_{r,i} = \frac{\sigma + |k_{r,i}| \sigma - \text{sgn}(e^T b x_{r,i}, S)}{1 - \sigma} \quad (i = 1, 2, ..., n) \]

This concludes the proof.

To accelerate the reaching phase and reduce chattering while maintaining sliding behaviour, an additional control signal, \( u_r \), is provided. When the state trajectories are far from the sliding hyperplane \( S = 0 \) is large), the switching gain should be correspondingly increased. When the state trajectories deviate from the sliding surface \( S = 0 \), if \( |S| \) is large the switching gain should be increased to force the trajectories back. When the state trajectories approach the sliding surface \( S = 0 \), if \( |S| \) is large the switching gain should be decreased to reduce chattering. Introducing the scaled modulus \( S \) and \( S_i \), the following fuzzy rules are proposed:

1. If \( S \) is large, then \( u_i \) is large
2. If \( S \) is small, then \( u_i \) is small
3. If \( S_i > 0 \) and \( S \) is large, then \( u_i \) is large
4. If \( S_i < 0 \) and \( S \) is large, then \( u_i \) is small
5. If \( S_i < 0 \) and \( S \) is small, then \( u_i \) is large
6. If \( S_i > 0 \) and \( S \) is small, then \( u_i \) is small

where \( u_i \) is a weighting factor depending on \( S \). Using exponential membership functions for \( S \) and \( S_i \), and singletons for \( u_i \) and \( u_r \), the centroid method for defuzzifying fuzzy schemes (iii)-(vi) gives

\[ u_{f1} = \frac{\mu_{S,large} u_{s1} + \mu_{S,small} 0}{\mu_{S,large} + \mu_{S,small}} \]  

\[ u_{f2} = \frac{\mu_{S,large} u_{s2} + \mu_{S,small} u_{s2}}{\mu_{S,large} + \mu_{S,small}} \]

where \( u_{s1} = \frac{1}{1 - \exp \left( -\frac{S}{\sigma_s} \right)} \) \( S > 0 \) and \( \exp \left( -\frac{S}{\sigma_s} \right) \) \( S < 0 \) for schemes (i)-(ii)

\[ u_r = \frac{\mu_{S,large} u_{r0} + \mu_{S,small} 0}{\mu_{S,large} + \mu_{S,small}} = u_{r0} \left( 1 - \exp \left( -\frac{S}{\sigma_r} \right) \right) \]

where \( u_{r0}, \sigma \) and \( \sigma_s \) are three parameters of the fuzzy controller.

**Application to a two-mass servosystem:** The proposed technique is applied to the control of a two mass servo-drive system with backlash nonlinearity. The plant model and specifications were given in [5]. The linearised state model of the plant can be written as

\[ \begin{bmatrix} T_m \\ T_{12} \\ T_{12} \\ \omega_0 \end{bmatrix} = \begin{bmatrix} -3.3 & -\alpha & 0 & 0 \\ 1.2 & 0 & 0 & 0 \\ 0 & 0.2 & 1.23 & 0 \\ 0 & 0.21 & 0 & 1.23 \end{bmatrix} \begin{bmatrix} T_m \\ \omega_1 \\ T_{12} \\ \omega_0 \end{bmatrix} + \begin{bmatrix} \alpha \\ 0 \\ 0 \omega_{ref} + 0 \omega_0 \end{bmatrix} \begin{bmatrix} T_L \omega_L \end{bmatrix} \]

where \( T_m, T_{12}, \) and \( \omega_0 \) are the motor torque, motor speed, elastic torque and load speed, respectively. \( \omega_{ref} \) is the reference input and external load torque, and \( \alpha = 237.3 \pm 18.7 \) is an uncertain element depending on the stiffness of the drive speed/torque characteristics. The hyperplane eigenvalues are chosen to be \( \lambda_{23} = \{-11 -11 -11\} \). The step response of the load speed is shown in Fig. 1 for the case of a 50% reduction in drive stiffness, backlash with a deadband width of \( 2 \varphi = 0.01 \), and a constant load torque \( T_L \) exerted at \( t = 5 \). Comparing with the responses obtained by pole placement (Fig. 1a), sliding mode control without tuning (Fig. 1b) indicates that an additional fuzzy tuner inherently improves system robustness.
References


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