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Decentralized H_∞ Control and Reliability Analysis for Symmetric Composite Systems: Dynamic Output Feedback Case

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Abstract. The dynamic output feedback decentralized H_∞ control and the reliability of the designed system (the maximal number of control-channel outages when the performance is still acceptable) is studied for symmetric composite systems. It is shown that the decentralized H_∞ control problem can be simplified to a simultaneous H_∞ control problem for two modified subsystems. A design method based on the simultaneous H_∞ control method is given. Simple methods for testing the reliability are presented using the special structure of symmetric composite systems.

Keywords. Symmetric composite systems, output feedback, decentralized control, fault tolerant control, reliability.

1 Introduction

Symmetric composite systems are those composed of identical subsystems which are symmetrically interconnected. The motivation for studying this class of systems is due to its very diverse application areas, such as in electric power systems, industrial manipulators, computer networks [7, 10, 12].

In recent years there has been a great interest in studying symmetric composite systems. It is shown that many analyses and synthesis problems for symmetric composite systems can be simplified because of their special structure. Lunze [10] first proposed the state-space model of symmetric composite systems, and investigated some fundamental properties of the systems. For centralized control problems, Liu [9] treated the output regulation for symmetric composite systems. The model reduction problem was considered by

Lam and Yang [11]. H_2 and H_∞ optimal control were studied in [7]. Yang *et al.* [14] and Huang *et al.* [6] analyzed the reliable control of such systems. For the decentralized control of symmetric composite systems, Lunze [10] proved that the system has no decentralized fixed modes if and only if it is completely controllable and observable. Sundareshan and Elbanna [12] presented a sufficient condition for such systems to be decentralized stabilizable using identical subsystem controllers. The decentralized control using two kinds of subsystem controllers is studied in [8]. The decentralized control for uncertain symmetric composite systems is studied in [2] and [15]. Furthermore, Bakule [1] considered the reduced-order decentralized control design of time-delayed uncertain symmetric composite systems.

In the last decade, a great deal of attention has been devoted to the H_∞ control of dynamic systems. As the H_∞ -norm is a particularly useful performance measure in solving such diverse control problems as disturbance rejection, model reference design, tracking, and robust design, many corresponding important design procedures have been established. An introduction of some standard results are found in [4].

Sometimes, control systems may result in unsatisfactory performance or even instability in the event of control component failures. Recently, Veillette *et al.* [13] considered the design of reliable control systems. The resulting control systems provide guaranteed stability and satisfy an H_∞ -norm disturbance attenuation bound not only when all control components are operational, but also in case of control-channel outages in the systems. The outages were restricted to occur within a preselected subset of available measurements or control inputs. Yang *et al.* [16] studied the reliable control using redundant controller and presented a simple design approach.

In [5], the state feedback fault tolerant decentralized H_∞ control problem for symmetric composite systems is studied. By using the special structure of the systems, a state feedback decentralized H_∞ control law is constructed by design state feedback H_∞ controllers for two modified subsystems. Since the full state of the control systems are generally not available in practice, to study the more general output feedback problem is of great important.

This paper is concerned with the dynamic output feedback decentralized H_∞ control and reliability analysis of symmetric composite systems. Firstly, it is shown that the dynamic output feedback decentralized H_∞ control problem is equivalent to a simultaneous H_∞ control problem of two modified subsystems. A design procedure based on the simultaneous H_∞ control method given in [3] is presented. The reliability considered in this paper concerns the largest number of control inputs or measurements failures that will keep the closed-loop system stable and maintain the required level of performance. By exploiting the special structure of the systems, it is shown that the computation of the poles and the H_∞ -norm of the resulting closed-loop system when control-channel outages occur can be reduced to the computation of the poles and the H_∞ -norm of three auxiliary systems. The order of one auxiliary system is twice that of any isolated subsystem, while the orders of

the other two auxiliary systems are equal to that of any isolated subsystem. Thus the tolerance to control input failures and measurement failures can be tested more efficiently.

The paper is organized as follows: Section 2 presents the problem formulation and preliminaries. The dynamic output feedback decentralized H_∞ control problem is studied and examples are given in Section 3. In Section 4, simple methods for testing the tolerance to control input failures and measurement failures are presented. The methods are also illustrated by the examples given in the previous section. Finally, a conclusion is given in Section 5.

2 Problem Formulation and Preliminaries

The symmetric composite system under consideration consists of N subsystems, the i th subsystem is described by

$$\begin{aligned} \dot{x}_i &= A_1 x_i + \sum_{k=1, k \neq i}^N A_2 x_k + B_{11} w_i + \sum_{k=1, k \neq i}^N B_{12} w_k + B_{21} u_i \\ z_i &= C_{11} x_i + D_{11} u_i \\ y_i &= C_{21} x_i + D_{21} w_i \end{aligned} \quad (1)$$

where $i = 1, 2, \dots, N$ and $x_i \in \mathbb{R}^n, u_i \in \mathbb{R}^m, w_i \in \mathbb{R}^r, z_i \in \mathbb{R}^s, y_i \in \mathbb{R}^p$ are the n, m, r, s and p dimensional state, control input, exogenous input, penalty and measured variables, respectively, and $A_1, A_2 \in \mathbb{R}^{n \times n}, B_{11}, B_{12} \in \mathbb{R}^{n \times r}, B_{21} \in \mathbb{R}^{n \times m}, C_{11} \in \mathbb{R}^{s \times n}, D_{11} \in \mathbb{R}^{s \times m}, C_{21} \in \mathbb{R}^{p \times n}, D_{21} \in \mathbb{R}^{p \times r}$.

Then the overall system is given by

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_1 u \\ y &= C_2 x + D_2 w \end{aligned} \quad (2)$$

where $x = [x_1^T, \dots, x_N^T]^T, u = [u_1^T, \dots, u_N^T]^T, w = [w_1^T, \dots, w_N^T]^T, z = [z_1^T, \dots, z_N^T]^T, y = [y_1^T, \dots, y_N^T]^T$ and $A \in \mathbb{R}^{Nn \times Nn}, B_1 \in \mathbb{R}^{Nn \times Nr}, B_2 \in \mathbb{R}^{Nn \times Nm}, C_1 \in \mathbb{R}^{Ns \times Nn}, D_1 \in \mathbb{R}^{Ns \times Nm}, C_2 \in \mathbb{R}^{Np \times Nn}, D_2 \in \mathbb{R}^{Np \times Nr}$ have the following structure

$$\begin{aligned} A &= \begin{bmatrix} A_1 & A_2 & \cdots & A_2 \\ A_2 & A_1 & \cdots & A_2 \\ \vdots & \vdots & \ddots & \vdots \\ A_2 & A_2 & \cdots & A_1 \end{bmatrix}, & B_1 &= \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{12} \\ B_{12} & B_{11} & \cdots & B_{12} \\ \vdots & \vdots & \ddots & \vdots \\ B_{12} & B_{12} & \cdots & B_{11} \end{bmatrix}, \\ B_2 &= \text{diag}[B_{21}, \dots, B_{21}], & C_1 &= \text{diag}[C_{11}, \dots, C_{11}], \\ D_1 &= \text{diag}[D_{11}, \dots, D_{11}], & C_2 &= \text{diag}[C_{21}, \dots, C_{21}], \\ D_2 &= \text{diag}[D_{21}, \dots, D_{21}]. \end{aligned}$$

The following dynamic output feedback decentralized H_∞ control problem is considered in the first part of this paper.

Decentralized H_∞ Control Problem (DHCP): Given $\gamma > 0$, find a decentralized dynamic output feedback controller

$$\begin{cases} \dot{\xi}_i = F_1 \xi_i + G_1 y_i \\ u_i = H_1 \xi_i \end{cases} \quad (3)$$

where $\xi_i \in \mathbb{R}^v$, $i = 1, \dots, N$, such that

(C1) the matrix $\begin{bmatrix} A & B_2 H \\ G C_2 & F \end{bmatrix}$ is stable, where

$$H = \text{diag}[H_1, \dots, H_1], \quad G = \text{diag}[G_1, \dots, G_1], \quad F = \text{diag}[F_1, \dots, F_1].$$

(C2) the transfer function matrix $T^c(s)$ of the closed-loop system

$$\begin{cases} \dot{x} = Ax + B_2 H \xi + B_1 w \\ \dot{\xi} = F \xi + G C_2 x + G D_2 w \\ z = C_1 x + D_1 H \xi \end{cases}$$

where $\xi = [\xi_1^T, \dots, \xi_N^T]^T$, satisfies $\|T^c(s)\|_\infty < \gamma$.

Remark 2.1 Note that the feedback controllers in (3) are identical for all subsystems, thus taking advantage of the structural properties of system (2) to reduce the complexity of the controller design.

The second part of this paper considers the reliability of the closed-loop system composed of system (2) and decentralized controller of the form (3). We will study the tolerance to control-channel outages, which include control input failures and measurement failures. It is supposed that the i th control input failure take the form $u_i = 0$ and the j th measurement failure is modelled as $y_j = 0$, $i, j \in \{1, \dots, N\}$.

The following notations and preliminaries are used in this note. For a matrix M , $\text{spec}(M)$ denotes the spectrum of M and $\rho(M)$ denotes the spectral radius of M . If not explicitly stated, I and 0 denote the identity matrix and the zero matrix of suitable dimension, respectively. Besides, all matrices are assumed to have compatible dimensions.

As in [5], for a positive integer p , we denote

$$m_k = [1 \ v_k \ v_k^2 \ \dots \ v_k^{p-1}]^T, \quad k = 1, 2, \dots, p$$

where $v_k = \exp(2\pi(k-1)\sqrt{-1}/p)$, $k = 1, 2, \dots, p$. That is, v_k is a root of the equation $v^p = 1$.

Denote

$$r_1 = m_1 = [1 \ 1 \ \dots \ 1]^T,$$

and

$$r_{\frac{p}{2}+1} = m_{\frac{p}{2}+1},$$

if p is an even number. Let $t = \frac{p+1}{2}$ if p is odd, $t = \frac{p}{2}$ if p is even. For $i = 2, 3, \dots, t$, define

$$r_i = \frac{1}{\sqrt{2}}(m_i + m_{p+2-i}), r_{p+2-i} = \frac{\sqrt{-1}}{\sqrt{2}}(m_i - m_{p+2-i}).$$

Further denote

$$R_p = \frac{1}{\sqrt{p}}[r_1 \ r_2 \ \cdots \ r_p] \quad (4)$$

$$T_{pi} = R_p \otimes I_i \quad (5)$$

where I_i is the $i \times i$ identity matrix and \otimes denotes the Kronecker product.

3 Decentralized H_∞ Control

3.1 Problem Simplification

In this note, we denote

$$A_\alpha = A_1 + (N-1)A_2, \quad A_\beta = A_1 - A_2, \quad (6)$$

$$B_{1\alpha} = B_{11} + (N-1)B_{12}, \quad B_{1\beta} = B_{11} - B_{12}, \quad (7)$$

then from Lemma 1 in [5] and (5), we have

$$\begin{cases} T_{Nn}^{-1}AT_{Nn} = \text{diag}[A_\alpha, A_\beta, \dots, A_\beta], \\ T_{Nn}^{-1}B_1T_{Nr} = \text{diag}[B_{1\alpha}, B_{1\beta}, \dots, B_{1\beta}], \quad T_{Nn}^{-1}B_2T_{Nm} = \text{diag}[B_{21}, \dots, B_{21}], \\ T_{Ns}^{-1}C_1T_{Nn} = \text{diag}[C_{11}, \dots, C_{11}], \quad T_{Np}^{-1}C_2T_{Nn} = \text{diag}[C_{21}, \dots, C_{21}], \\ T_{Ns}^{-1}D_1T_{Nm} = \text{diag}[D_{11}, \dots, D_{11}], \quad T_{Np}^{-1}D_2T_{Nr} = \text{diag}[D_{21}, \dots, D_{21}]. \end{cases} \quad (8)$$

We further denote

$$A^c = \begin{bmatrix} A & B_2H \\ GC_2 & F \end{bmatrix}, \quad A_\alpha^c = \begin{bmatrix} A_\alpha & B_{21}H_1 \\ G_1C_{21} & F_1 \end{bmatrix}, \quad A_\beta^c = \begin{bmatrix} A_\beta & B_{21}H_1 \\ G_1C_{21} & F_1 \end{bmatrix}, \quad (9)$$

$$W = sI - A^c, \quad W_\alpha = sI - A_\alpha^c, \quad W_\beta = sI - A_\beta^c, \quad (10)$$

then the following lemma holds.

Lemma 3.1 *There exists a permutation matrix P such that*

$$P^{-1} \begin{bmatrix} T_{Nn}^{-1} & 0 \\ 0 & T_{Nv}^{-1} \end{bmatrix} W^{-1} \begin{bmatrix} T_{Nn} & 0 \\ 0 & T_{Nv} \end{bmatrix} P = \text{diag}[W_\alpha^{-1}, W_\beta^{-1}, \dots, W_\beta^{-1}].$$

Proof: Let P be the permutation matrix such that

$$\begin{aligned} & P^{-1} \begin{bmatrix} \text{diag}[A_\alpha, A_\beta, \dots, A_\beta] & \text{diag}[B_{21}H_1, \dots, B_{21}H_1] \\ \text{diag}[G_1C_{21}, \dots, G_1C_{21}] & \text{diag}[F_1, \dots, F_1] \end{bmatrix} P \\ = & \text{diag} \left\{ \begin{bmatrix} A_\alpha & B_{21}H_1 \\ G_1C_{21} & F_1 \end{bmatrix}, \begin{bmatrix} A_\beta & B_{21}H_1 \\ G_1C_{21} & F_1 \end{bmatrix}, \dots, \begin{bmatrix} A_\beta & B_{21}H_1 \\ G_1C_{21} & F_1 \end{bmatrix} \right\}. \end{aligned}$$

Then from (8), we have

$$\begin{aligned} & P^{-1} \begin{bmatrix} T_{Nn}^{-1} & 0 \\ 0 & T_{Nv}^{-1} \end{bmatrix} W^{-1} \begin{bmatrix} T_{Nn} & 0 \\ 0 & T_{Nv} \end{bmatrix} P \\ = & \left\{ P^{-1} \begin{bmatrix} T_{Nn}^{-1} & 0 \\ 0 & T_{Nv}^{-1} \end{bmatrix} W \begin{bmatrix} T_{Nn} & 0 \\ 0 & T_{Nv} \end{bmatrix} P \right\}^{-1} \\ = & \left\{ P^{-1} \begin{bmatrix} \text{diag}[sI - A_\alpha, sI - A_\beta, \dots, sI - A_\beta] \\ \text{diag}[-G_1C_{21}, \dots, -G_1C_{21}] \\ \text{diag}[-B_{21}H_1, \dots, -B_{21}H_1] \\ \text{diag}[sI - F_1, \dots, sI - F_1] \end{bmatrix} P \right\}^{-1} \\ = & \left\{ \text{diag} \left\{ \begin{bmatrix} sI - A_\alpha & -B_{21}H_1 \\ -G_1C_{21} & sI - F_1 \end{bmatrix}, \begin{bmatrix} sI - A_\beta & -B_{21}H_1 \\ -G_1C_{21} & sI - F_1 \end{bmatrix}, \right. \right. \\ & \left. \left. \dots, \begin{bmatrix} sI - A_\beta & -B_{21}H_1 \\ -G_1C_{21} & sI - F_1 \end{bmatrix} \right\} \right\}^{-1} \\ = & \text{diag} [W_\alpha^{-1}, W_\beta^{-1}, \dots, W_\beta^{-1}]. \end{aligned}$$

The proof is completed. \blacksquare

Using the special structure of system (2), the DHCP can be simplified to a simultaneously H_∞ control problem for two modified subsystems, as shown in the following theorem.

Theorem 3.2 *A decentralized dynamic output feedback controller of the form (3) satisfies conditions (C1) and (C2) if and only if it satisfies the following two conditions:*

(C1)' A_α^c and A_β^c are stable.

(C2)' $\|T_\alpha^c(s)\|_\infty < \gamma$ and $\|T_\beta^c(s)\|_\infty < \gamma$, where

$$T_\alpha^c(s) = \begin{bmatrix} C_{11} & D_{11}H_1 \end{bmatrix} W_\alpha^{-1} \begin{bmatrix} B_{1\alpha} \\ G_1D_{21} \end{bmatrix}$$

and

$$T_\beta^c(s) = \begin{bmatrix} C_{11} & D_{11}H_1 \end{bmatrix} W_\beta^{-1} \begin{bmatrix} B_{1\beta} \\ G_1D_{21} \end{bmatrix}. \quad (11)$$

Proof: (C1) \iff (C1)': From (8), we have

$$\begin{aligned}
& \text{spec} \left\{ \begin{bmatrix} A & B_2 H \\ GC_2 & F \end{bmatrix} \right\} \\
= & \text{spec} \left\{ \begin{bmatrix} T_{Nn}^{-1} & 0 \\ 0 & T_{Nv}^{-1} \end{bmatrix} \begin{bmatrix} A & B_2 H \\ GC_2 & F \end{bmatrix} \begin{bmatrix} T_{Nn} & 0 \\ 0 & T_{Nv} \end{bmatrix} \right\} \\
= & \text{spec} \left\{ \begin{bmatrix} T_{Nn}^{-1} A T_{Nn} & T_{Nn}^{-1} B_2 T_{Nm} T_{Nm}^{-1} H T_{Nv} \\ T_{Nv}^{-1} G T_{Np} T_{Np}^{-1} C_2 T_{Nn} & T_{Nv}^{-1} F T_{Nv} \end{bmatrix} \right\} \\
= & \text{spec} \left\{ \begin{bmatrix} \text{diag}[A_\alpha, A_\beta, \dots, A_\beta] & \text{diag}[B_{21} H_1, \dots, B_{21} H_1] \\ \text{diag}[G_1 C_{21}, \dots, G_1 C_{21}] & \text{diag}[F_1, \dots, F_1] \end{bmatrix} \right\} \\
= & \text{spec} \left\{ \text{diag} \left\{ \begin{bmatrix} A_\alpha & B_{21} H_1 \\ G_1 C_{21} & F_1 \end{bmatrix}, \begin{bmatrix} A_\beta & B_{21} H_1 \\ G_1 C_{21} & F_1 \end{bmatrix}, \right. \right. \\
& \left. \left. \dots, \begin{bmatrix} A_\beta & B_{21} H_1 \\ G_1 C_{21} & F_1 \end{bmatrix} \right\} \right\}.
\end{aligned}$$

Hence (C1) holds if and only if (C1)' holds.

(C2) \iff (C2)': From (5), it is easy to see that T_{pi} is an orthogonal matrix for integers $p \geq 2$ and $i \geq 1$. Since $T^c(s) = [C_1 \quad D_1 H] W^{-1} \begin{bmatrix} B_1 \\ GD_2 \end{bmatrix}$, premultiplication or postmultiplication of $T^c(s)$ by orthogonal matrices will leave the H_∞ -norm unchanged. Hence we have

$$\begin{aligned}
\|T^c(s)\|_\infty &= \|T_{Ns}^{-1} T^c(s) T_{Nr}\|_\infty \\
&= \left\| T_{Ns}^{-1} [C_1 \quad D_1 H] \begin{bmatrix} T_{Nn} & 0 \\ 0 & T_{Nv} \end{bmatrix} P P^{-1} \begin{bmatrix} T_{Nn}^{-1} & 0 \\ 0 & T_{Nv}^{-1} \end{bmatrix} W^{-1} \right. \\
&\quad \left. \times \begin{bmatrix} T_{Nn} & 0 \\ 0 & T_{Nv} \end{bmatrix} P P^{-1} \begin{bmatrix} T_{Nn}^{-1} & 0 \\ 0 & T_{Nv}^{-1} \end{bmatrix} \begin{bmatrix} B_1 \\ GD_2 \end{bmatrix} T_{Nr} \right\|_\infty.
\end{aligned}$$

From Lemma 3.1,

$$\begin{aligned}
\|T^c(s)\|_\infty &= \left\| \begin{bmatrix} \text{diag}[C_{11}, \dots, C_{11}] & \text{diag}[D_{11} H_1, \dots, D_{11} H_1] \end{bmatrix} P \right. \\
&\quad \left. \times \text{diag}[W_\alpha^{-1}, W_\beta^{-1}, \dots, W_\beta^{-1}] P^{-1} \begin{bmatrix} \text{diag}[B_{1\alpha}, B_{1\beta}, \dots, B_{1\beta}] \\ \text{diag}[G_1 D_{21}, \dots, G_1 D_{21}] \end{bmatrix} \right\|_\infty \\
&= \left\| \text{diag} \left\{ [C_{11} \quad D_{11} H_1], \dots, [C_{11} \quad D_{11} H_1] \right\} \right. \\
&\quad \left. \times \text{diag}[W_\alpha^{-1}, W_\beta^{-1}, \dots, W_\beta^{-1}] \right. \\
&\quad \left. \times \text{diag} \left\{ \begin{bmatrix} B_{1\alpha} \\ G_1 D_{21} \end{bmatrix}, \begin{bmatrix} B_{1\beta} \\ G_1 D_{21} \end{bmatrix}, \dots, \begin{bmatrix} B_{1\beta} \\ G_1 D_{21} \end{bmatrix} \right\} \right\|_\infty \\
&= \left\| \text{diag}[T_\alpha^c(s), T_\beta^c(s), \dots, T_\beta^c(s)] \right\|_\infty \\
&= \max \left\{ \|T_\alpha^c(s)\|_\infty, \|T_\beta^c(s)\|_\infty \right\}.
\end{aligned}$$

Thus (C2) holds if and only if (C2)' holds. \blacksquare

Remark 3.3 Theorem 3.2 shows that the DHCP is equivalent to the simultaneous H_∞ control problem of two modified subsystems

$$\begin{cases} \dot{x}_i = A_i x_i + B_{1i} w_i + B_{21} u \\ z_i = C_{11} x_i + D_{11} u \\ y = C_{21} x_i + D_{21} w_i \end{cases} \quad (i = \alpha, \beta) \quad (12)$$

using the same controller

$$\begin{cases} \dot{\eta} = F_1 \eta + G_1 y \\ u = H_1 \eta \end{cases} . \quad (13)$$

The two systems (12) have the same order as any isolated subsystem of system (2), hence they are referred to as the modified subsystems.

3.2 Controller Design

In the following, we make some standard assumptions:

- (i) $(A_\alpha, B_{1\alpha}), (A_\alpha, B_{21}), (A_\beta, B_{1\beta}), (A_\beta, B_{21})$ are all stabilizable;
- (ii) $(C_{11}, A_\alpha), (C_{21}, A_\alpha), (C_{11}, A_\beta), (C_{21}, A_\beta)$ are all detectable;
- (iii) $D_{11}^T \begin{bmatrix} C_{11} & D_{11} \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix}$;
- (iv) $\begin{bmatrix} B_{1\alpha} \\ D_{21} \end{bmatrix} D_{21}^T = \begin{bmatrix} 0 \\ I \end{bmatrix}$, $\begin{bmatrix} B_{1\beta} \\ D_{21} \end{bmatrix} D_{21}^T = \begin{bmatrix} 0 \\ I \end{bmatrix}$.

The assumptions are equivalent to those usually applied to (2) in a standard H_∞ synthesis problem and that lead to the following well known result [4].

Lemma 3.4 [4] *For $\gamma > 0$ and $i = \alpha, \beta$, there exists an admissible controller such that $\|T_i^c(s)\|_\infty < \gamma$ if and only if the following three conditions hold*

(S1) *there exists a $X_{i\infty} \geq 0$, such that*

$$A_i^T X_{i\infty} + X_{i\infty} A_i + X_{i\infty} (\gamma^{-2} B_{1i} B_{1i}^T - B_{21} B_{21}^T) X_{i\infty} + C_{11}^T C_{11} = 0 \quad (14)$$

(S2) *there exists a $Y_{i\infty} \geq 0$, such that*

$$A_i Y_{i\infty} + Y_{i\infty} A_i^T + Y_{i\infty} (\gamma^{-2} C_{11}^T C_{11} - C_{21}^T C_{21}) Y_{i\infty} + B_{1i} B_{1i}^T = 0 \quad (15)$$

(S3) $\rho(Y_{i\infty} X_{i\infty}) < \gamma^2$.

Defining

$$\begin{aligned}
 F_{i\infty} &= B_{21}^T X_{i\infty} \\
 L_{i\infty} &= (I - \gamma^{-2} Y_{i\infty} X_{i\infty})^{-1} Y_{i\infty} C_{21}^T \\
 \hat{A}_{i\infty} &= A_i + \gamma^{-2} B_{1i} B_{1i}^T X_{i\infty} - B_{21} F_{i\infty} - L_{i\infty} C_{21} \\
 Z_{i\infty} &= (I - \gamma^{-2} Y_{i\infty} X_{i\infty})^{-1}
 \end{aligned} \tag{16}$$

then all admissible controllers $K_{i\infty}(Q_i(s))$ resulting in $\|T_i^c(s)\|_\infty < \gamma$, are parameterized as in Figure 1, where

$$M_{i\infty}(s) \longleftrightarrow \left[\begin{array}{c|cc} \hat{A}_{i\infty} & L_{i\infty} & Z_{i\infty} B_{21} \\ \hline -F_{i\infty} & 0 & I \\ -C_{21} & I & 0 \end{array} \right]$$

and $Q_i(s)$ is a stable real-rational transfer function satisfying $\|Q_i(s)\|_\infty < \gamma$.

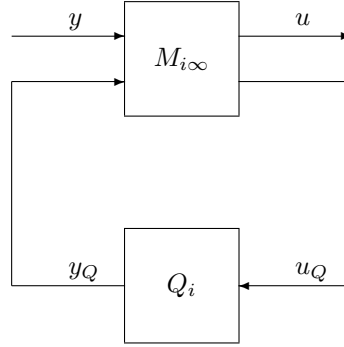


Figure 1

From Theorem 3.2 and Lemma 3.4, the necessary conditions for the DHCP to have a solution are (S1)-(S3) hold for $i = \alpha, \beta$. And if these conditions hold, the problem is to find $Q_\alpha(s)$ and $Q_\beta(s)$ such that

$$\|Q_\alpha(s)\|_\infty < \gamma, \quad \|Q_\beta(s)\|_\infty < \gamma$$

and

$$K_{\alpha\infty}(Q_\alpha(s)) = K_{\beta\infty}(Q_\beta(s)).$$

Such a problem was considered in [3], from Theorem 2 in [3], we have the following theorem.

Theorem 3.5 For $\gamma > 0$, if there exist two matrices $N_\alpha \geq 0, N_\beta \geq 0$, such that

$$\begin{aligned}
 N_\alpha \tilde{A}_\alpha^T + \tilde{A}_\alpha N_\alpha + N_\alpha \tilde{E}_\alpha N_\alpha + \tilde{R}_\alpha \tilde{R}_\alpha^T &= 0, \\
 N_\beta \tilde{A}_\beta^T + \tilde{A}_\beta N_\beta + N_\beta \tilde{E}_\beta N_\beta + \tilde{R}_\beta \tilde{R}_\beta^T &= 0
 \end{aligned} \tag{17}$$

where

$$\begin{aligned}\tilde{A}_\alpha &= \begin{bmatrix} \hat{A}_{\alpha\infty} + Z_{\alpha\infty}B_{21}F_{\alpha\infty} & Z_{\alpha\infty}B_{21}F_{\beta\infty} - L_{\alpha\infty}C_{21} \\ 0 & \hat{A}_{\beta\infty} + L_{\beta\infty}C_{21} \end{bmatrix} \\ \tilde{A}_\beta &= \begin{bmatrix} \hat{A}_{\alpha\infty} + L_{\alpha\infty}C_{21} & 0 \\ Z_{\beta\infty}B_{21}F_{\alpha\infty} - L_{\beta\infty}C_{21} & \hat{A}_{\beta\infty} + Z_{\beta\infty}B_{21}F_{\beta\infty} \end{bmatrix} \\ \tilde{E}_\alpha &= \tilde{E}_\beta = \begin{bmatrix} \gamma^{-2}F_{\alpha\infty}^T F_{\alpha\infty} - C_{21}^T C_{21} & \gamma^{-2}F_{\alpha\infty}^T F_{\beta\infty} - C_{21}^T C_{21} \\ \gamma^{-2}F_{\beta\infty}^T F_{\alpha\infty} - C_{21}^T C_{21} & \gamma^{-2}F_{\beta\infty}^T F_{\beta\infty} - C_{21}^T C_{21} \end{bmatrix} \\ \tilde{R}_\alpha &= \begin{bmatrix} -\frac{1}{2}L_{\alpha\infty} \\ \frac{1}{2}L_{\beta\infty} \end{bmatrix} + N_\beta \begin{bmatrix} C_{21}^T \\ C_{21}^T \end{bmatrix} \end{aligned} \quad (18)$$

$$\tilde{R}_\beta = \begin{bmatrix} \frac{1}{2}L_{\alpha\infty} \\ -\frac{1}{2}L_{\beta\infty} \end{bmatrix} + N_\alpha \begin{bmatrix} C_{21}^T \\ C_{21}^T \end{bmatrix} \quad (19)$$

then the controller

$$\begin{cases} \dot{\xi}_i = (\hat{A} - \hat{L}\bar{C})\xi_i + \hat{L}y_i \\ u_i = \hat{F}\xi_i \end{cases} \quad (20)$$

is a solution of the DHCP, where $i = 1, \dots, N$,

$$\begin{aligned}\hat{A} &= \begin{bmatrix} \hat{A}_{\alpha\infty} + L_{\alpha\infty}C_{21} & 0 \\ 0 & \hat{A}_{\beta\infty} + L_{\beta\infty}C_{21} \end{bmatrix}, \\ \hat{L} &= \begin{bmatrix} \frac{1}{2}L_{\alpha\infty} \\ \frac{1}{2}L_{\beta\infty} \end{bmatrix} + (N_\alpha + N_\beta) \begin{bmatrix} C_{21}^T \\ C_{21}^T \end{bmatrix}, \\ \hat{F} &= [-F_{\alpha\infty}, -F_{\beta\infty}], \\ \bar{C} &= [C_{21}, C_{21}]. \end{aligned}$$

Proof: From Theorem 2 in [3], if (17) holds, then the controller

$$\begin{cases} \dot{\eta} = (\hat{A} - \hat{L}\bar{C})\eta + \hat{L}y \\ u = \hat{F}\eta \end{cases}$$

is a simultaneous H_∞ suboptimal controller that guarantees $\|T_\alpha^c(s)\|_\infty < \gamma$ and $\|T_\beta^c(s)\|_\infty < \gamma$. From Theorem 3.2, (20) is a solution of the DHCP. ■

Remark 3.6 As shown in [3], the parameter embedding method may be used to solve the two equations in (17). Alternatively, we may use the following iterative method: First let $N_\alpha^{(0)} = N_\beta^{(0)} = 0$ in (18) and (19), then the two equations in (17) can be solved. Using the obtained solutions $N_\alpha^{(1)}$ and $N_\beta^{(1)}$ to calculate \tilde{R}_α and \tilde{R}_β and solve for $N_\alpha^{(2)}$ and $N_\beta^{(2)}$. The process is continued until $N_\alpha^{(k)}$ and $N_\beta^{(k)}$ converge.

3.3 Examples

Example 1. Consider the voltage/reactive power behaviour of multimachine power system consisting of several synchronous machines including their PI-voltage controllers, which feed the load through a distribution net [10] with a system model given by

$$\begin{aligned}
 \dot{x}_i &= \begin{bmatrix} -2.51 & -0.16 \\ 2.55 & 0 \end{bmatrix} x_i + \sum_{k=1, k \neq i}^N \begin{bmatrix} -0.065 & 0 \\ -0.0027 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0.9 \\ -1 \end{bmatrix} u_i \\
 &+ \begin{bmatrix} 0 & 0.02 \\ 0 & 0.01 \end{bmatrix} w_i + \sum_{k=1, k \neq i}^N \begin{bmatrix} 0 & 0.01 \\ 0 & 0.01 \end{bmatrix} w_k \\
 z_i &= \begin{bmatrix} 2 & 0.2 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i \\
 y_i &= \begin{bmatrix} 2.54 & 0 \end{bmatrix} x_i + \begin{bmatrix} 1 & 0 \end{bmatrix} w_i \quad (i = 1, \dots, 20)
 \end{aligned} \tag{21}$$

Suppose $\gamma = 0.62$, it is easy to test that assumptions (i)-(iv) hold and hence Theorem 3.5 can be applied to design the controller. Using the iterative method suggested in Remark 3.6 to solve the two equations in (17), we obtain the decentralized H_∞ controller

$$\begin{cases} \dot{\xi}_i = \begin{bmatrix} -2.4451 & -0.1597 & -0.0003 & 0 \\ 2.5527 & -0.0002 & 0.0002 & 0 \\ -0.0247 & 0 & -3.6915 & -0.0484 \\ 0.1767 & 0 & 2.6478 & -0.0400 \end{bmatrix} \xi_i + \begin{bmatrix} 0.0001 \\ -0.0001 \\ 0.0097 \\ -0.0696 \end{bmatrix} y_i \\ u_i = \begin{bmatrix} 0.0002 & 0.0002 & 0.0549 & 0.0788 \end{bmatrix} \xi_i \end{cases} \tag{22}$$

where $i = 1, \dots, 20$. The resulting closed-loop system is stable and satisfies the performance requirement. In fact,

$$\|T_\alpha^c(s)\|_\infty = 0.6143 < \gamma, \quad \|T_\beta^c(s)\|_\infty = 0.0613 < \gamma.$$

Example 2. Consider open-loop unstable symmetric composite system composed of $N = 20$ subsystems given by

$$\begin{aligned}
 \dot{x}_i &= \begin{bmatrix} -2 & -6 \\ 5 & 0 \end{bmatrix} x_i + \sum_{k=1, k \neq i}^N \begin{bmatrix} -0.500 & 0.1 \\ -0.002 & 0.1 \end{bmatrix} x_k + \begin{bmatrix} 2 \\ -1 \end{bmatrix} u_i \\
 &+ \begin{bmatrix} 0 & 0.02 \\ 0 & 0.01 \end{bmatrix} w_i + \sum_{k=1, k \neq i}^N \begin{bmatrix} 0 & 0.01 \\ 0 & 0.01 \end{bmatrix} w_k \\
 z_i &= \begin{bmatrix} 2 & 0.2 \\ 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_i \\
 y_i &= \begin{bmatrix} 2 & 3 \end{bmatrix} x_i + \begin{bmatrix} 1 & 0 \end{bmatrix} w_i \quad (i = 1, \dots, 20)
 \end{aligned} \tag{23}$$

Suppose $\gamma = 10$, it is easy to test that assumptions (i)-(iv) hold and hence Theorem 3.5 can be applied to design the controller. Using the iterative method suggested in Remark 3.6 to solve the two equations in (17), we obtain the decentralized H_∞ controller

$$\left\{ \begin{array}{l} \dot{\xi}_i = \begin{bmatrix} -10.2465 & 0.3761 & 0.1692 & 0.2538 \\ 3.8375 & -1.0802 & -0.5880 & -0.8821 \\ 0.0086 & 0.0130 & -4.1214 & -4.8844 \\ 0.0077 & 0.0115 & 6.3247 & -0.6898 \end{bmatrix} \xi_i + \begin{bmatrix} -0.0846 \\ 0.2940 \\ -0.0043 \\ -0.0038 \end{bmatrix} y_i \\ u_i = [0.5402 \quad 2.1067 \quad -1.3150 \quad 0.6013] \xi_i \end{array} \right. \quad (24)$$

where $i = 1, \dots, 20$. To test the effectiveness of the controller, we compute $\text{spec}(A_\alpha^c)$, $\text{spec}(A_\beta^c)$, $\|T_\alpha^c(s)\|_\infty$, $\|T_\beta^c(s)\|_\infty$ and obtain

$$\begin{aligned} \text{spec}(A_\alpha^c) &= \{-9.7727, -10.4685, -0.1682, -0.5199, -2.4043 \pm 5.2885i\} \subset \mathbb{C}^-, \\ \text{spec}(A_\beta^c) &= \{-10.3945, -0.2453, -1.1333 \pm 5.4437i, -2.4157 \pm 5.2913i\} \subset \mathbb{C}^-, \\ \|T_\alpha^c(s)\|_\infty &= 4.9213 < \gamma, \quad \|T_\beta^c(s)\|_\infty = 2.4335 < \gamma. \end{aligned}$$

4 Reliability Analysis

In this section, we consider the reliability of the closed-loop system composed of system (2) and decentralized controller of the form (3). Here the reliability means the ability of tolerate control channel failures such as control-input channel failure or sensor measurement channel failures.

For integer $1 \leq l \leq N - 1$, we denote

$$\begin{aligned} U_l &= A_1 + (l - 1)A_2, \quad \tilde{U}_l = \sqrt{l(N - l)}A_2. \\ V_l &= B_{11} + (l - 1)B_{12}, \quad \tilde{V}_l = \sqrt{l(N - l)}B_{12}. \end{aligned}$$

4.1 Control Input Failures

Since the subsystems (1) are all identical, without loss of generality, we may assume that the first l control inputs fail. Then the system matrix of the resulting closed-loop system is

$$A_l^c = \begin{bmatrix} A & (B_2H)_l \\ GC_2 & F \end{bmatrix}$$

where

$$(B_2H)_l = \text{diag} \left[\overbrace{0, \dots, 0}^l, \overbrace{B_{21}H_1, \dots, B_{21}H_1}^{N-l} \right].$$

The transfer function matrix of the resulting closed-loop system is

$$T_l^c(s) = [C_1 \quad (D_1H)_l] (sI - A_l^c)^{-1} \begin{bmatrix} B_1 \\ GD_2 \end{bmatrix}$$

where

$$(D_1H)_l = \text{diag} \left[\overbrace{0, \dots, 0}^l, \overbrace{D_{11}H_1, \dots, D_{11}H_1}^{N-l} \right].$$

Using a similar method as in the proof of Theorem 4 in [5], the following theorem can be established.

Theorem 4.1 *When l control inputs fail, the poles of the resulting closed-loop system is*

$$\text{spec}(A_l^c) = \begin{cases} \text{spec}(\bar{A}_l^c) \cup \text{spec}(A_\beta^c) & (l = 1) \\ \text{spec}(\bar{A}_l^c) \cup \text{spec}(\bar{A}_\beta^c) \cup \text{spec}(A_\beta^c) & (2 \leq l \leq N - 2) \\ \text{spec}(\bar{A}_l^c) \cup \text{spec}(\bar{A}_\beta^c) & (l = N - 1) \end{cases} \quad (25)$$

where

$$\bar{A}_l^c = \begin{bmatrix} U_l & \tilde{U}_l & 0 & 0 \\ \tilde{U}_l & U_{N-l} & 0 & B_{21}H_1 \\ G_1C_{21} & 0 & F_1 & 0 \\ 0 & G_1C_{21} & 0 & F_1 \end{bmatrix}, \quad \bar{A}_\beta^c = \begin{bmatrix} A_\beta & 0 \\ G_1C_{21} & F_1 \end{bmatrix}$$

and A_β^c is defined in (9). The transfer function matrix of the resulting closed-loop system is

$$T_l^c(s) = \begin{bmatrix} T_{ls} & 0 \\ 0 & T_{(N-l)s} \end{bmatrix} \text{diag} \left[\bar{T}_l^c(s), \overbrace{\bar{T}_\beta^c(s), \dots, \bar{T}_\beta^c(s)}^{l-1}, \overbrace{T_\beta^c(s), \dots, T_\beta^c(s)}^{N-l-1} \right] \\ \times \begin{bmatrix} T_{lr}^{-1} & 0 \\ 0 & T_{(N-l)r}^{-1} \end{bmatrix} \quad (26)$$

where

$$\bar{T}_l^c(s) = \begin{bmatrix} C_{11} & 0 & 0 & 0 \\ 0 & C_{11} & 0 & D_{11}H_1 \end{bmatrix} (sI - \bar{A}_l^c)^{-1} \begin{bmatrix} V_l & \tilde{V}_l \\ \tilde{V}_l & V_{N-l} \\ G_1D_{21} & 0 \\ 0 & G_1D_{21} \end{bmatrix},$$

$$\bar{T}_\beta^c(s) = [C_{11} \quad 0] (sI - \bar{A}_\beta^c)^{-1} \begin{bmatrix} B_{1\beta} \\ G_1D_{21} \end{bmatrix}$$

and $T_\beta^c(s)$ is defined in (11). Moreover, if A_l^c is stable, the H_∞ -norm of the transfer function matrix is

$$\|T_l^c(s)\|_\infty = \begin{cases} \max \left\{ \|\bar{T}_l^c(s)\|_\infty, \|T_\beta^c(s)\|_\infty \right\} & (l = 1) \\ \max \left\{ \|\bar{T}_l^c(s)\|_\infty, \|\bar{T}_\beta^c(s)\|_\infty, \|T_\beta^c(s)\|_\infty \right\} & (2 \leq l \leq N - 2) \\ \max \left\{ \|\bar{T}_l^c(s)\|_\infty, \|\bar{T}_\beta^c(s)\|_\infty \right\} & (l = N - 1) \end{cases}$$

4.2 General Failures

In practice, a particular *control-channel failure* may have three possibilities: control-input failure only, measurement failure only, and simultaneous control-input and measurement failure. The following theorem shows that the above three possibilities can be considered together.

Theorem 4.2 *For $1 \leq l \leq N - 1$, the poles of the resulting closed-loop system for any l control-channel failures are the same. Moreover, the transfer function matrices are also identical.*

Proof: Suppose there are $1 \leq l \leq N - 1$ control-channel failures in which l_1 control-channels with only control-input failures, l_2 control-channels with only measurement failures, and l_3 control-channels with both control-input and measurement failures, thus $l_1 + l_2 + l_3 = l$. The system matrix and the transfer function matrix of the resulting closed-loop system are

$$\check{A}_l^c = \begin{bmatrix} A & B_2 H_* \\ G_* C_2 & F \end{bmatrix}$$

and

$$\check{T}_l^c(s) = [C_1 \quad D_1 H_*] (sI - \check{A}_l^c)^{-1} \begin{bmatrix} B_1 \\ G_* D_2 \end{bmatrix}$$

respectively, where

$$G_* = \text{diag} \left[\overbrace{G_1, \dots, G_1}^{l_1}, \overbrace{0, \dots, 0}^{l_2}, \overbrace{0, \dots, 0}^{l_3}, \overbrace{G_1, \dots, G_1}^{N-l} \right]$$

$$H_* = \text{diag} \left[\overbrace{0, \dots, 0}^{l_1}, \overbrace{H_1, \dots, H_1}^{l_2}, \overbrace{0, \dots, 0}^{l_3}, \overbrace{H_1, \dots, H_1}^{N-l} \right].$$

When $sI - F$ is invertible, we have

$$\begin{aligned} |sI - \check{A}_l^c| &= \begin{vmatrix} sI - A & -B_2 H_* \\ -G_* C_2 & sI - F \end{vmatrix} \\ &= |sI - F| |(sI - A) - B_2 H_* (sI - F)^{-1} G_* C_2|. \end{aligned}$$

Since

$$(sI - F)^{-1} = \text{diag} [(sI - F_1)^{-1}, \dots, (sI - F_1)^{-1}],$$

we have

$$= \text{diag} \left[\begin{array}{c} B_2 H_* (sI - F)^{-1} G_* C_2 \\ \underbrace{0, \dots, 0}_l, \overbrace{B_{21} H_1 (sI - F_1)^{-1} G_1 C_{21}, \dots, B_{21} H_1 (sI - F_1)^{-1} G_1 C_{21}}^{N-l} \end{array} \right]$$

which is only affected by $l = l_1 + l_2 + l_3$. Thus

$$\text{spec}(\check{A}_l^c) = \text{spec}(A_l^c).$$

Moreover, since system (2) has the particular symmetric structure and the decentralized controller (3) have identical subsystem controllers, measurement failures and control-input failures have the same effect on the closed-loop transfer function matrix [13], that is

$$\check{T}_l^c(s) = T_l^c(s).$$

The proof is completed. ■

Remark 4.3 Since l measurement failures is a special case of l control-channel failures, the results of l measurement failures are the same as that of l control-input failures.

Remark 4.4 It can be easily seen from (25) that the necessary condition for A_l^c to be stable ($l \geq 1$) is $\text{spec}(F_1) \subset \mathbb{C}^-$. In other words, $\text{spec}(F_1) \subset \mathbb{C}^-$ is necessary for the closed-loop system to endure at least one control-channel failure.

Remark 4.5 From Theorems 4.1 and 4.2, one can determine the reliability of the controller by simply computing the poles and the H_∞ -norm of at most three lower order systems. Noting that there is no conservativeness introduced in the simplification, thus the resulting reliability is also exact.

Remark 4.6 The control channel failure considered in this paper means the control channel of particular subsystems completely fail. If each subsystem is a multiple-input system and the same part of the input fail for l subsystems, similar reliability analysis results can be obtained. However, if different part of the input fail in different subsystems, the result in this paper cannot be applied because the symmetric structure of the whole system will no longer hold. Certainly the reliability analysis method suitable for arbitrary system could still be applied but the analysis will not be so simple.

4.3 Examples

Example 1. Consider the closed-loop system composed of system (21) and the controller (22) designed in Section 3.3. For $1 \leq l \leq 19$ control-channel failures, Theorem 4.1 is used to compute $\|T_l^c(s)\|_\infty$. The results are summarized in Table 1 ($l = 0$ refers to the closed-loop system with no control-channel failure).

As in Section 3.3, we take $\gamma = 0.62$, Table 1 shows that for $1 \leq l \leq 16$, $\|T_l^c(s)\|_\infty < \gamma$ while $\|T_{17}^c(s)\|_\infty > \gamma$. Hence the closed-loop system will maintain its stability and with the norm of the transfer function matrix less than γ when $l < 16$ control-channel failures occur.

Table 1: Summary of results for Example 1 ($\gamma = 0.62$)

l	$\ T_l^c(s)\ _\infty$	l	$\ T_l^c(s)\ _\infty$	l	$\ T_l^c(s)\ _\infty$	l	$\ T_l^c(s)\ _\infty$
0	0.6143	5	0.6160	10	0.6177	15	0.6195
1	0.6146	6	0.6164	11	0.6181	16	0.6198
2	0.6150	7	0.6167	12	0.6184	17	0.6201
3	0.6153	8	0.6170	13	0.6188	18	0.6205
4	0.6157	9	0.6174	14	0.6191	19	0.6208

Example 2. Consider the closed-loop system composed of system (23) and the controller (24) designed in Section 3.3. For $1 \leq l \leq 19$ control-channel failures, Theorem 4.1 is used to compute $\text{spec}(A_l^c)$ and $\|T_l^c(s)\|_\infty$. The results are summarized in Table 1 ($l = 0$ and $l = 20$ refer to the closed-loop system with no control-channel failure and the open-loop system, respectively).

As in Section 3.3, we take $\gamma = 10$, Table 2 shows that for $1 \leq l \leq 8$, A_l^c is stable and $\|T_l^c(s)\|_\infty < \gamma$ while $\|T_9^c(s)\|_\infty > \gamma$. Hence the closed-loop system will maintain its stability and with the norm of the transfer function matrix less than γ when $l < 9$ control-channel failures occur. Moreover, the closed-loop system becomes unstable when $l \geq 14$ control-channel failures occur.

Table 2: Summary of results for Example 2 ($\gamma = 10$)

l	A_l^c	$\ T_l^c(s)\ _\infty$	l	A_l^c	$\ T_l^c(s)\ _\infty$	l	A_l^c	$\ T_l^c(s)\ _\infty$
0	stable	4.9213	7	stable	8.3385	14	unstable	/
1	stable	5.1988	8	stable	9.4292	15	unstable	/
2	stable	5.5178	9	stable	10.9398	16	unstable	/
3	stable	5.8893	10	stable	13.1864	17	unstable	/
4	stable	6.3286	11	stable	16.9131	18	unstable	/
5	stable	6.8579	12	stable	24.3899	19	unstable	/
6	stable	7.5104	13	stable	47.4609	20	unstable	/

5 Conclusion

This paper discussed the dynamic output feedback decentralized H_∞ control and reliability analysis of symmetric composite systems. By using the

structural properties of the systems, we give a simple method to design its dynamic output feedback decentralized H_∞ controller. Moreover, the reliability of the controller can be easily tested by computing the poles and the H_∞ -norm of systems of possibly much lower orders. It should be noted that no conservativeness were introduced in the simplification process, which is a major difference between the method used in this paper and other methods for testing reliability.

Though we provide simple controller design and reliability analysis methods for symmetric composite systems, the reliability analysis can only be conducted after the controller design. In practice, the method proposed in this paper can be applied in an iterative way. That is, if the reliability of the designed system is not satisfactory, then we can increase the performance requirement for the nominal case design and use our method to check the reliability again. How to design the controller such that it possesses a predefined level of reliability is our future research work.

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