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Robust H_∞ reliable control for a class of uncertain neutral delay systems

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This paper deals with the problem of robust reliable control for a class of uncertain neutral delay systems. The aim was to design a state feedback controller such that the plant remained stable for all admissible uncertainties as well as actuator faults among a prespecified subset of actuators or sector-type actuator non-linearity, independently of the delay time. A linear matrix inequality approach was developed to solve the problem addressed with an H_∞ norm bound constraint on disturbance attenuation.

1. Introduction

In practical applications, since faults of control components often occur, an important reliability requirement for the design of reliable systems is to guarantee the stability and basic performance of the plant by a single controller which can tolerate sensor or actuator faults. That is, the essential stability and performance requirements for the control systems remain achieved, not only when the systems operating properly, but also in the presence of certain system measurement or control input faults. This motivates the development of the so-called reliable control theory. More specifically, actuators are very important in transforming the controller output to the plant. Therefore, the question of how to preserve the closed-loop control system performance in face of actuator faults will be both difficult and meaningful. Such a problem has attracted great attention in recent years (Veillette *et al*. 1992, Seo and Kim 1996).

While actuator faults represent a drastic change in the system structure, another common source of non-linearities associated with actuators is sector-type actuator non-linearity. This arises from the physical limitations of the actuator driven by the signal generated from the designed controller. Sector-type actuator non-linearity not only deteriorates the performance of control systems, but also can lead to instability. If sector non-linearity is not taken into account in the design of the control system, integrator wind-up or limit cycle may occur (Glattfelder and Schaufelberger 1983, Kosut 1983). Therefore, the stability analysis and stabilization of linear systems with sector-type actuator nonlinearity have been investigated by many researchers over the past two decades (Glattfelder and Schaufelberger 1983, Chen and Wang 1988).

On the other hand, it is well known that dynamic systems with time-delay are common in chemical process, electrical heater and long transmission lines in pneumatic, hydraulic and rolling mill systems. The stability and stabilization of time-delay systems is a problem of both practical and theoretical interest since the existence of a delay in a physical system often induces instability or poor performance. Thus, the research on time-delay systems has attracted many researchers. Considerable research efforts have been undertaken for the past four decades on various aspects of dynamical systems with delays in the state.

There are also numerous control systems depending not only on state delays, but also on the derivatives of the delayed state. Such systems are referred to as neutral delay systems (Hale 1977, Dugard and Verriest 1998). Issues such as stability and stabilization, relating to neutral delay systems have been studied (Chu 1997, Dugard and Verriot 1998, Hui and Hu 1997, Logeman and Townley 1996, Park and Won 1999). The stabilization of neutral delay systems with actuator saturation was given in Tarboureich and Garcia (1999). However, due to the complexity of neutral delay systems, there is very

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little result on the reliable control for neutral delay systems, that is, the preservation of performance under actuator faults or saturation.

This paper considers two kinds of robust reliable control problem for a class of uncertain neutral delay systems. The first case is for actuator faults among a prespecified subset of actuators, the other case is for sector-type actuator non-linearity. Attention is focused on the design of state feedback controllers that guarantee, for all admissible uncertainties as well as for actuator faults or sector type actuator non-linearity, closed-loop systems asymptotic stability with an H_∞ norm bound constraint on disturbance attenuation, independently of the delay time. It will be shown that the problem addressed can be solved in terms of linear matrix inequality (LMI), and the resulting neutral delay control systems provide guaranteed robust reliable stability despite of possible actuator faults or sector type actuator non-linearity.

Throughout, $\mathbb{R}^n$ denotes the real n-dimensional linear vector space. The matrix I denotes an identity matrix of appropriate dimensions. $W > 0$ (< 0) denotes a positive-definite (negative-definite) real symmetric matrix.

2. Robust reliable control: actuator faults case

2.1. *Problem Formulation*

Consider the following uncertain neutral delay system:

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (A_h + \Delta A_h(t))x(t-h) + (A_d + \Delta A_d(t))\,\dot{x}(t-d) + Bu(t) + D\omega(t) \quad (1)$$

$$z(t) = Cx(t) \quad (2)$$

$$x(\theta) = \Phi(\theta), \qquad \forall\theta \in [-\tau, 0], \quad (3)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $\omega(t) \in \mathbb{R}^q$ is the disturbance input which belongs to $L_2[0,\infty)$ and $z(t) \in \mathbb{R}^s$ is the controlled output. Here, A, A_h, B, A_d, D and C are known constant matrices with appropriate dimensions. $\Delta A(t)$, $\Delta A_h(t)$ and $\Delta A_d(t)$ are real-valued matrix functions representing parameter uncertainties. $d > 0, h > 0$ denote time-delays. $\tau = \max\{h, d\}$ and $\Phi(\theta)$ are continuously differentiable vector-valued initial functions on $[-\tau, 0]$.

Suppose the uncertain structure of system from (1) to (3) is given by

$$[\Delta A(t) \quad \Delta A_h(t) \quad \Delta A_d(t)] = FE(t)[H_1 \quad H_2 \quad H_3],$$

where F, H_1, H_2 and H_3 are known constant matrices with appropriate dimensions and $E(t)$ is an unknown matrix function of uncertain parameters satisfying

$$E^{\mathrm{T}}(t)E(t) \le I.$$

Actuators of a given system can be classified into two sets. The set of actuators that are susceptible to faults is denoted as $\Omega \subseteq \{1, 2, \ldots, m\}$, and is possible to fail. The complementary set of actuators that are robust to faults and essential to stabilize a given system is therefore denoted as $\bar{\Omega} = \{1, 2, \ldots, m\}\backslash\Omega$, and is assumed never to fail. As pointed out in Seo and Kim (1996), the actuators belonging to Ω are redundant when considering system stabilization but are useful for improving control system performance, while the actuators belonging to $\bar{\Omega}$ are needed to stabilize the given system, no matter whether $\bar{\Omega}$ is a minimum set. Introduce the decomposition

$$B = [B_\Omega \quad B_{\bar{\Omega}}],$$

where B_Ω is the control matrix associated with the set Ω, and $B_{\bar{\Omega}}$ is the control matrix associated with the complementary subset of control inputs. It is assumed that $(A, B_{\bar{\Omega}})$ is stabilizable.

Actuators play an important role in transmitting the controller output to the plant. Generally, the outputs of the failed actuators may have arbitrary signals different from normal controller outputs, and these signals will act on the system as unexpected control inputs. Here, the output of a failed actuator belongs to $L_2[0,\infty)$. It is desirable that the effects of actuator faults can be reduced by feedback, and the stability of closed-loop system is maintained. Therefore, the outputs of the faulty actuators are regarded as disturbance inputs. Attempts are made to suppress the signals on the system outputs caused by faulty actuators as well as disturbance inputs below a given level.

Let $\varphi \subseteq \Omega$ correspond to a particular subset of the actuators that actually experience a fault, and assume that the actuator faults are modelled as $u_\varphi(t)$ whose elements correspond to the set of faulty actuators φ. We adopt the following notation, which will be used in the derivation of the main result

$$B = [\,B_\varphi \quad B_{\bar{\varphi}}\,],$$

where $\bar{\varphi} = \bar{\Omega} \cup \Omega\backslash\varphi$, B_φ and $B_{\bar{\varphi}}$ have meanings analogous to those of B_Ω and $B_{\bar{\Omega}}$. Since

$$\begin{aligned} B_{\bar{\varphi}}B_{\bar{\varphi}}^{\mathrm{T}} &= [B_{\bar{\Omega}} \quad B_{\Omega\backslash\varphi}]([B_{\bar{\Omega}} \quad B_{\Omega\backslash\varphi}])^{\mathrm{T}} \\ &= B_{\bar{\Omega}}B_{\bar{\Omega}}^{\mathrm{T}} + B_{\Omega\backslash\varphi}B_{\Omega\backslash\varphi}^{\mathrm{T}} \\ B_\Omega B_\Omega^{\mathrm{T}} &= [B_\varphi \quad B_{\Omega\backslash\varphi}]([B_\varphi \quad B_{\Omega\backslash\varphi}])^{\mathrm{T}} \\ &= B_\varphi B_\varphi^{\mathrm{T}} + B_{\Omega\backslash\varphi}B_{\Omega\backslash\varphi}^{\mathrm{T}} \end{aligned}$$

it follows that

$$B_{\bar{\Omega}}B_{\bar{\Omega}}^{\mathrm{T}} \le B_{\bar{\varphi}}B_{\bar{\varphi}}^{\mathrm{T}}, \quad B_\varphi B_\varphi^{\mathrm{T}} \le B_\Omega B_\Omega^{\mathrm{T}}. \quad (4)$$

Denote

$$\bar{\omega}(t) = \begin{bmatrix} \omega(t) \\ u_\varphi(t) \end{bmatrix},$$

where $\bar{\omega}(t)$ can be considered as the new disturbance input vector. System from (1) to (3) with state feedback control law

$$u(t) = Lx(t) \tag{5}$$

becomes

$$\dot{x}(t) = (A + \Delta A(t) + BL)x(t) + (A_h + \Delta A_h(t))x(t-h) + (A_d + \Delta A_d(t))\,\dot{x}(t-d) + D\omega(t) \tag{6}$$

$$z(t) = Cx(t). \tag{7}$$

To obtain the compatible dimension of B_Ω and $B_{\bar{\Omega}}$, the feedback gain is decomposed

$$L = \begin{bmatrix} L_\Omega \\ L_{\bar{\Omega}} \end{bmatrix} = \begin{bmatrix} L_\varphi \\ L_{\bar{\varphi}} \end{bmatrix}.$$

Therefore, we have

$$u(t) = \begin{bmatrix} u_\varphi(t) \\ L_{\bar{\varphi}}x(t) \end{bmatrix}.$$

The system in (6) and (7) may be rewritten as

$$\dot{x}(t) = \bar{A}_\Delta x(t) + (A_h + \Delta A_h(t))x(t-h) + (A_d + \Delta A_d(t))\,\dot{x}(t-d) + \bar{D}\bar{\omega}(t) \tag{8}$$

$$z(t) = Cx(t), \tag{9}$$

where

$$\bar{A}_\Delta = A + \Delta A(t) + B_{\bar{\varphi}}L_{\bar{\varphi}}$$
$$\bar{D} = [D \quad B_\varphi].$$

The robust reliable control problem due to actuator faults is stated in the following.

Problem RRCAF: To determine a state feedback law (5) such that the following requirements are met.

P1 The closed-loop system from (8) to (9) is asymptotically stable.

P2 H_∞ norm bound constraint on the disturbance attenuation for all admissible uncertainties and relevant actuator faults is guaranteed. That is, under zero initial conditions,

$$\|z(t)\|_2 \le \gamma \|\bar{\omega}(t)\|_2,$$

where $\gamma > 0$ is a given constant.

2.2. *Main result*

This section presents a method for designing a robust and reliable H_∞ controller to ensure that the closed-loop system from (8) to (9) is asymptotically stable with disturbance attenuation γ, for all admissible uncertainties as well as actuator faults among a prespecified subset of actuators.

Some lemmas will be used to establish our main results. The result in Lemma 3 has also appeared in Xu *et al.* (2001), but the proof given below is essentially different.

Lemma 1 (Li and de Souza 1997, Xu *et al.* 2001)**:** *Let A, F, H, $E(t)$, and P be real matrices of appropriate dimensions with $P > 0$ and $E(t)$ satisfying*

$$E^{\mathrm{T}}(t)E(t) \le I.$$

Then we have the following results:

(1) *For any $\varepsilon > 0$,*

$$FE(t)H + (FE(t)H)^{\mathrm{T}} \le \varepsilon^{-1}FF^{\mathrm{T}} + \varepsilon H^{\mathrm{T}}H.$$

(2) *For any $\varepsilon > 0$ such that*

$$\varepsilon I - HPH^{\mathrm{T}} > 0$$

it follows that

$$(A + FE(t)H)P(A + FE(t)H)^{\mathrm{T}} \le APA^{\mathrm{T}} + APH^{\mathrm{T}}(\varepsilon I - HPH^{\mathrm{T}})^{-1}HPA^{\mathrm{T}} + \varepsilon FF^{\mathrm{T}}$$

or equivalently

$$(A + FE(t)H)P(A + FE(t)H)^{\mathrm{T}} \le AP(P - \varepsilon^{-1}PH^{\mathrm{T}}HP)^{-1}PA^{\mathrm{T}} + \varepsilon FF^{\mathrm{T}}$$

(3)

$$AF + (AF)^{\mathrm{T}} \le APA^{\mathrm{T}} + F^{\mathrm{T}}P^{-1}F$$

Lemma 2 (Xu *et al.* 2001): *Consider neutral delay system from* (1) *to* (3) *without uncertainty and $u(t) = 0$, $\omega(t) = 0$, given by*

$$\dot{x}(t) = Ax(t) + A_h x(t-h) + A_d \dot{x}(t-d) \tag{10}$$

$$x(\theta) = \Phi(\theta), \qquad \forall\theta \in [-\tau, 0]. \tag{11}$$

If there exist matrices $P > 0$, $Q > 0$ and $S > 0$ such that the following LMI holds:

$$\begin{bmatrix} PA^{\mathrm{T}} + AP + Q + S & (Q + S + PA^{\mathrm{T}})A_d^{\mathrm{T}} & PA_h^{\mathrm{T}} \\ A_d(Q + S + AP) & A_d(Q + S)A_d^{\mathrm{T}} - Q & 0 \\ A_hP & 0 & -S \end{bmatrix} < 0$$

or

$$\begin{bmatrix} PA + A^{\mathrm{T}}P + Q + S & (Q + S + PA)A_d & PA_h \\ A_d^{\mathrm{T}}(Q + S + A^{\mathrm{T}}P) & A_d^{\mathrm{T}}(Q + S)A_d - Q & 0 \\ A_h^{\mathrm{T}}P & 0 & -S \end{bmatrix} < 0$$

Lemma 3 (Xu *et al.* 2001): *Consider neutral delay system from* (1) *to* (3) *without uncertainty and* $u(t) = 0$. *That is,*

$$\dot{x}(t) = Ax(t) + A_h x(t-h) + A_d \dot{x}(t-d) + D\omega(t) \tag{12}$$

$$z(t) = Cx(t) \tag{13}$$

$$x(\theta) = \Phi(\theta), \qquad \forall \theta \in [-\tau, 0]. \tag{14}$$

If there exist matrices $P > 0$, $Q > 0$ *and* $S > 0$ *such that the following LMI holds:*

$$\begin{bmatrix} PA + A^{\mathrm{T}}P + Q + S + C^{\mathrm{T}}C & (PA + Q + S + C^{\mathrm{T}}C)A_d & PA_h & PD \\ A_d^{\mathrm{T}}(PA + Q + S + C^{\mathrm{T}}C)^{\mathrm{T}} & -M_1 & 0 & 0 \\ A_h^{\mathrm{T}}P & 0 & -S & 0 \\ D^{\mathrm{T}}P & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{15}$$

or

$$\begin{bmatrix} PA^{\mathrm{T}} + AP + Q + S + DD^{\mathrm{T}} & (PA^{\mathrm{T}} + Q + S + DD^{\mathrm{T}})A_d^{\mathrm{T}} & PA_h^{\mathrm{T}} & PC^{\mathrm{T}} \\ A_d(PA^{\mathrm{T}} + Q + S + DD^{\mathrm{T}})^{\mathrm{T}} & -\bar{M}_1 & 0 & 0 \\ A_h P & 0 & -S & 0 \\ CP & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0, \tag{16}$$

then system from (10)–(11) *is asymptotically stable independently of the delays h and d.*
where

$$M_1 = Q - A_d^{\mathrm{T}}(C^{\mathrm{T}}C + Q + S)A_d$$

$$\bar{M}_1 = Q - A_d(DD^{\mathrm{T}} + Q + S)A_d^{\mathrm{T}}$$

then system from (12) *to* (14) *is asymptotically stable with disturbance attenuation* γ *independently of the delays h and d.*

Proof: By Lemma 2 and LMI (15), the asymptotic stability of the system from (12) to (14) can be easily deduced.

Now we consider the disturbance attenuation problem. A Lyapunov functional candidate for system from (12) to (14) is introduced as

$$\begin{aligned} V(x_t) &= (x(t) - A_d x(t-d))^{\mathrm{T}} P (x(t) - A_d x(t-d)) \\ &\quad + \int_{-h}^{0} x^{\mathrm{T}}(t+\theta) S x(t+\theta)\,\mathrm{d}\theta \\ &\quad + \int_{-d}^{0} x^{\mathrm{T}}(t+\theta) Q x(t+\theta)\,\mathrm{d}\theta \end{aligned} \tag{17}$$

where

$$x_t = x(t+\theta), \quad \theta \in [-\tau, 0].$$

Decide an associated Hamiltonian $H(x(t), \omega(t), t)$ as follows:

$$H(x(t), \omega(t), t) = \dot{V}(x_t) + z^{\mathrm{T}}(t)z(t) - \gamma^2 \omega^{\mathrm{T}}(t)\omega(t). \tag{18}$$

Thus, under zero initial conditions, we have

$$\begin{aligned} &\int_0^{\mathrm{T}} (z^{\mathrm{T}}(t)z(t) - \gamma^2 \omega^{\mathrm{T}}(t)\omega(t))\,\mathrm{d}t \\ &\quad = \int_0^{\mathrm{T}} (\dot{V}(x_t) + z^{\mathrm{T}}(t)z(t) - \gamma^2 \omega^{\mathrm{T}}(t)\omega(t))\,\mathrm{d}t - V(x_T) \\ &\quad = \int_0^{\mathrm{T}} H(x(t), \omega(t), t)\,\mathrm{d}t - V(x_T). \end{aligned}$$

Since the asymptotic stability of system from (12) to (14) has been obtained, we can see that Lemma 3 holds if

$$H(x(t), \omega(t), t) < 0,$$

which implies

$$\int_0^{\infty} z^{\mathrm{T}}(t)z(t)\,\mathrm{d}t - \gamma^2 \int_0^{\infty} \omega^{\mathrm{T}}(t)\omega(t)\,\mathrm{d}t < 0.$$

By differentiating (17) along the trajectories of system from (12) to (14), we get

$$\begin{aligned} &H(x(t), \omega(t), t) \\ &\quad = 2(x(t) - A_d x(t-d))^{\mathrm{T}} P (Ax(t) \\ &\qquad + A_h x(t-h) + D\omega(t)) + x^{\mathrm{T}}(t)(S+Q)x(t) \\ &\qquad - x^{\mathrm{T}}(t-d) Q x(t-d) - x^{\mathrm{T}}(t-h) S x(t-h) \\ &\qquad + z^{\mathrm{T}}(t)z(t) - \gamma^2 \omega^{\mathrm{T}}(t)\omega(t) \\ &\quad = (x(t) - A_d x(t-d))^{\mathrm{T}} (PA + A^{\mathrm{T}}P + S + Q + C^{\mathrm{T}}C) \\ &\qquad \times (x(t) - A_d x(t-d)) \\ &\qquad - x^{\mathrm{T}}(t-d) M_1 x(t-d) + 2(x(t) - A_d x(t-d))^{\mathrm{T}} \\ &\qquad \times (S + Q + C^{\mathrm{T}}C + PA) A_d x(t-d) \end{aligned}$$

$$+ 2(x(t) - A_d x(t-d))^{\mathrm{T}} P A_h x(t-h) + 2(x(t) - A_d x(t-d))^{\mathrm{T}} P D\omega(t) - x^{\mathrm{T}}(t-h) S x(t-h) - \gamma^2 \omega^{\mathrm{T}}(t)\omega(t). \quad (19)$$

Define a difference operator $\mathcal{D}$ as

$$\mathcal{D}(\phi(t)) = \phi(t) - A_d \phi(t-d).$$

Noting the definition of the operator $\mathcal{D}$, (19) can be rewritten as

$$\begin{aligned} H(x(t),\omega(t),t) &= \mathcal{D}^{\mathrm{T}}(x(t))(PA + A^{\mathrm{T}}P + S + Q + C^{\mathrm{T}}C) \\ &\times \mathcal{D}(x(t)) + 2\mathcal{D}^{\mathrm{T}}(x(t))(S+Q+C^{\mathrm{T}}C + PA)A_d x(t-d) \\ &+ 2\mathcal{D}^{\mathrm{T}}(x(t))PA_h x(t-h) + 2\mathcal{D}^{\mathrm{T}}(x(t))PD\omega(t) \\ &- x^{\mathrm{T}}(t-d)M_1(t-d) - x^{\mathrm{T}}(t-h)Sx(t-h) \\ &- \gamma^2\omega^{\mathrm{T}}(t)\omega(t). \end{aligned}$$

By Lemma 1, it follows that

$$\begin{aligned} H(x(t),\omega(t),t) &\le \mathcal{D}^{\mathrm{T}}(x(t))(PA + A^{\mathrm{T}}P + Q + S + C^{\mathrm{T}}C \\ &+ (PA + Q + S + C^{\mathrm{T}}C)A_d M_1^{-1} A_d^{\mathrm{T}} \\ &\times (PA + Q + S + C^{\mathrm{T}}C)^{\mathrm{T}} + PA_h S^{-1} A_h^{\mathrm{T}} P \\ &+ \gamma^{-2} PDD^{\mathrm{T}} P)\mathcal{D}(x(t)). \end{aligned}$$

From LMI (15), we can obtain that

$$H(x(t),\omega(t),t) < 0.$$

Similarly, LMI (16) can be easily obtained following the above method. Thus the proof of Lemma 3 is completed. ∎

From the above lemmas, we can prove the main result of this section.

Theorem 1: *If there exist matrices $X > 0$, $Q > 0$, $S > 0$ and scalars $\varepsilon_i > 0$, $i = 1, \ldots, 5$, such that the following LMI holds,*

$$\begin{bmatrix} W_1 & 0 & XA_h^{\mathrm{T}} & XC^{\mathrm{T}} & L_1 & 0 \\ 0 & W_2 & 0 & 0 & 0 & L_2 \\ A_h X & 0 & W_3 & 0 & 0 & 0 \\ CX & 0 & 0 & -\gamma^2 I & 0 & 0 \\ L_1^{\mathrm{T}} & 0 & 0 & 0 & -J_1 & 0 \\ 0 & L_2^{\mathrm{T}} & 0 & 0 & 0 & -J_2 \end{bmatrix} < 0, \quad (20)$$

where

$$\begin{aligned} W_1 &= XA^{\mathrm{T}} + AX + Q + S + DD^{\mathrm{T}} - 2B_{\bar{\Omega}}B_{\bar{\Omega}}^{\mathrm{T}} + B_{\Omega}B_{\Omega}^{\mathrm{T}} + \varepsilon_1 FF^{\mathrm{T}} \\ W_2 &= \varepsilon_5 FF^{\mathrm{T}} + A_d(Q + S + DD^{\mathrm{T}} + B_{\Omega}B_{\Omega}^{\mathrm{T}})A_d^{\mathrm{T}} - Q \\ W_3 &= -S + \varepsilon_2 FF^{\mathrm{T}} \\ L_1 &= [\, XH_1^{\mathrm{T}} \quad XH_2^{\mathrm{T}} \quad XH_1^{\mathrm{T}} \quad Q + S + DD^{\mathrm{T}} + B_{\Omega}B_{\Omega}^{\mathrm{T}} + XA^{\mathrm{T}} \,] \\ L_2 &= [\, F \quad A_d \quad A_d(Q + S + DD^{\mathrm{T}} + B_{\Omega}B_{\Omega}^{\mathrm{T}})H_3^{\mathrm{T}} \,] \\ J_1 &= \mathrm{diag}[\, \varepsilon_1 I \quad \varepsilon_2 I \quad \varepsilon_3 I \quad I - \varepsilon_3 FF^{\mathrm{T}} \,] \\ J_2 &= \mathrm{diag}[\, \varepsilon_4 I \quad I - \varepsilon_4 H_3^{\mathrm{T}} H_3 \quad \varepsilon_5 I - H_3(Q + S + DD^{\mathrm{T}} + B_{\Omega}B_{\Omega}^{\mathrm{T}})H_3^{\mathrm{T}} \,]. \end{aligned}$$

Then the state feedback control law

$$u(t) = Lx(t), \quad L = -B^{\mathrm{T}}X^{-1} \quad (21)$$

robustly stabilizes the uncertain neutral delay system from (1) *to* (3) *with disturbance attenuation γ, independently of the time delays h and d, for all admissible uncertainties as well as all actuator faults corresponding to $\varphi \in \Omega$.*

Proof: In the case of actuator faults, applying controller (21) to neutral delay system from (1) to (3), the resulting closed-loop system can be described as (8) to (9). Under the conditions of Theorem 1, we notice that $\varepsilon_i > 0$, $i = 3, 4, 5$ satisfy

$$\begin{aligned} I - \varepsilon_3 FF^{\mathrm{T}} &> 0 \\ I - \varepsilon_4 H_3^{\mathrm{T}} H_3 &> 0 \\ \varepsilon_5 I - H_3(Q + S + DD^{\mathrm{T}} + B_{\Omega}B_{\Omega}^{\mathrm{T}})H_3^{\mathrm{T}} &> 0. \end{aligned}$$

Then by Lemma 1, we have the following inequalities:

$$\begin{aligned} X\bar{A}_{\Delta}^{\mathrm{T}} + \bar{A}_{\Delta}X &= XA^{\mathrm{T}} + AX + X\Delta A^{\mathrm{T}}(t) + \Delta A(t)X - 2B_{\bar{\varphi}}B_{\bar{\varphi}}^{\mathrm{T}} \\ &\le XA^{\mathrm{T}} + AX + X\Delta A^{\mathrm{T}}(t) + \Delta A(t)X - 2B_{\bar{\Omega}}B_{\bar{\Omega}}^{\mathrm{T}} \\ &\le XA^{\mathrm{T}} + AX - 2B_{\bar{\Omega}}B_{\bar{\Omega}}^{\mathrm{T}} + \varepsilon_1^{-1} XH_1^{\mathrm{T}} H_1 X + \varepsilon_1 FF^{\mathrm{T}} \end{aligned} \quad (22)$$

$$\begin{bmatrix} 0 & X\Delta A_h^{\mathrm{T}}(t) \\ \Delta A_h(t)X & 0 \end{bmatrix} \le \begin{bmatrix} \varepsilon_2^{-1} XH_2^{\mathrm{T}} H_2 X & 0 \\ 0 & \varepsilon_2 FF^{\mathrm{T}} \end{bmatrix} \quad (23)$$

$$
\begin{bmatrix} 0 & (Q+S+\bar{D}\bar{D}^{\mathrm{T}}+X(A+\Delta A(t))^{\mathrm{T}})(A_d+\Delta A_d(t))^{\mathrm{T}} \\ (A_d+\Delta A_d(t))(Q+S+\bar{D}\bar{D}^{\mathrm{T}}+X(A+\Delta A(t))^{\mathrm{T}})^{\mathrm{T}} & 0 \end{bmatrix}
$$
$$
\leq \begin{bmatrix} (Q+S+\bar{D}\bar{D}^{\mathrm{T}}+X(A+\Delta A(t))^{\mathrm{T}})(Q+S+\bar{D}\bar{D}^{\mathrm{T}}+X(A+\Delta A(t))^{\mathrm{T}})^{\mathrm{T}} & 0 \\ 0 & (A_d+\Delta A_d(t))(A_d+\Delta A_d(t))^{\mathrm{T}} \end{bmatrix}
$$
$$
\leq \begin{bmatrix} Q_1(I-\varepsilon_3 FF^{\mathrm{T}})^{-1}Q_1^{\mathrm{T}}+\varepsilon_3^{-1}XH_1^{\mathrm{T}}H_1X & 0 \\ 0 & A_d(I-\varepsilon_4 H_3^{\mathrm{T}}H_3)^{-1}A_d^{\mathrm{T}}+\varepsilon_4^{-1}FF^{\mathrm{T}} \end{bmatrix}, \tag{24}
$$

where

$$
Q_1 = Q+S+DD^{\mathrm{T}}+B_\Omega B_\Omega^{\mathrm{T}}+XA^{\mathrm{T}}
$$

and

$$
\begin{aligned}
&(A_d+\Delta A_d(t))(Q+S+\bar{D}\bar{D}^{\mathrm{T}})(A_d+\Delta A_d(t))^{\mathrm{T}} \\
&\leq A_d(Q+S+DD^{\mathrm{T}}+B_\Omega B_\Omega^{\mathrm{T}})A_d^{\mathrm{T}}+\varepsilon_5 FF^{\mathrm{T}} \\
&\quad + A_d(Q+S+DD^{\mathrm{T}}+B_\Omega B_\Omega^{\mathrm{T}})H_3^{\mathrm{T}}\left[\varepsilon_5 I - H_3(Q+S+DD^{\mathrm{T}}+B_\Omega B_\Omega^{\mathrm{T}})H_3^{\mathrm{T}}\right]^{-1} \\
&\quad \times H_3(Q+S+DD^{\mathrm{T}}+B_\Omega B_\Omega^{\mathrm{T}})A_d^{\mathrm{T}}.
\end{aligned} \tag{25}
$$

By Schur complements, it follows from (20) and the inequalities (22)–(25) that

$$
\begin{bmatrix} X\bar{A}_\Delta^{\mathrm{T}}+\bar{A}_\Delta X+Q+S+\bar{D}\bar{D}^{\mathrm{T}} & (Q+S+\bar{D}\bar{D}^{\mathrm{T}}+X\bar{A}_\Delta^{\mathrm{T}})(A_d+\Delta A_d(t))^{\mathrm{T}} & X(A_h+\Delta A_h(t))^{\mathrm{T}} & XC^{\mathrm{T}} \\ (A_d+\Delta A_d(t))(Q+S+\bar{D}\bar{D}^{\mathrm{T}}+X\bar{A}_\Delta^{\mathrm{T}})^{\mathrm{T}} & (A_d+\Delta A_d(t))(Q+S+\bar{D}\bar{D}^{\mathrm{T}})(A_d+\Delta A_d(t))^{\mathrm{T}}-Q & 0 & 0 \\ (A_h+\Delta A_h(t))X & 0 & -S & 0 \\ CX & 0 & 0 & -\gamma^2 I \end{bmatrix}
$$
$$
< 0
$$

Thus, the closed-loop system in (8) and (9) is robustly stable with disturbance attenuation γ according to Lemma 3. ■

If the term associated with $\dot{x}(t-d)$ does not appear in system from (1)–(3), Theorem 1 reduces to the following corollary.

Corollary 1: *Consider the uncertain system given by*

$$
\begin{aligned}
\dot{x}(t) &= (A+\Delta A(t))x(t)+(A_h+\Delta A_h(t))x(t-h) \\
&\quad + Bu(t)+D\omega(t)
\end{aligned} \tag{26}
$$
$$
z(t) = Cx(t) \tag{27}
$$
$$
x(\theta)=\Phi(\theta), \qquad \forall\theta\in[-h,0] \tag{28}
$$

If there exist matrices $X>0$, $S>0$ and scalars $\varepsilon_i>0$, $i=1,2$, such that the following LMI holds.

$$
\begin{bmatrix} W_1 & XA_h^{\mathrm{T}} & XC^{\mathrm{T}} & XH_1^{\mathrm{T}} & XH_2^{\mathrm{T}} \\ A_hX & W_2 & 0 & 0 & 0 \\ CX & 0 & -\gamma^2 I & 0 & 0 \\ H_1X & 0 & 0 & -\varepsilon_1 I & 0 \\ H_2X & 0 & 0 & 0 & -\varepsilon_2 I \end{bmatrix} < 0,
$$

where

$$
\begin{aligned}
W_1 &= XA^{\mathrm{T}}+AX+S+DD^{\mathrm{T}}+B_\Omega B_\Omega^{\mathrm{T}} \\
&\quad - 2B_{\bar{\Omega}}B_{\bar{\Omega}}^{\mathrm{T}}+\varepsilon_1 FF^{\mathrm{T}} \\
W_2 &= -S+\varepsilon_2 FF^{\mathrm{T}},
\end{aligned}
$$

then the state feedback control law (21) *robustly stabilizes uncertain delay system from* (26) *to* (28) *with disturbance attenuation γ, independently of the time delay h, for all admissible uncertainties as well as all actuator faults corresponding to $\varphi\in\Omega$.*

Furthermore, when there is no time delays in system from (1) to (3), Theorem 1 reduces to the result as stated in following corollary without proof.

Corollary 2: *Consider uncertain system*

$$
\dot{x}(t) = (A+\Delta A(t))x(t)+Bu(t)+D\omega(t) \tag{29}
$$
$$
z(t) = Cx(t) \tag{30}
$$
$$
x(0) = x_0. \tag{31}
$$

If there exist matrix $X>0$ and scalar $\varepsilon>0$ such that the following LMI or inequality holds.

$$\begin{bmatrix} XA^{\mathrm{T}}+AX+DD^{\mathrm{T}}+B_{\Omega}B_{\Omega}^{\mathrm{T}} \\ \quad -2B_{\bar{\Omega}}B_{\Omega}^{\mathrm{T}}+\varepsilon FF^{\mathrm{T}} & XC^{\mathrm{T}} & XH_1^{\mathrm{T}} \\ CX & -\gamma^2 I & 0 \\ H_1X & 0 & -\varepsilon I \end{bmatrix} < 0$$

or

$$A^{\mathrm{T}}X+XA+\gamma^{-2}X(DD^{\mathrm{T}}+B_{\Omega}B_{\Omega}^{\mathrm{T}})X+\varepsilon XFF^{\mathrm{T}}X - 2XB_{\bar{\Omega}}B_{\Omega}^{\mathrm{T}}X+\varepsilon^{-1}H_1^{\mathrm{T}}H_1+C^{\mathrm{T}}C<0,$$

then the state feedback control law (21) *robustly stabilizes uncertain system from* (29) *to* (31) *with disturbance attenuation* γ, *for all admissible uncertainties as well as all actuator faults corresponding to* $\varphi \in \Omega$.

Remark 1: The problem in Corollary 1 was considered in (Wang 1998, Theorem 1). In Corollary 1, solutions can be obtained by solving an LMI, which is easier than solving a modified algebraic Riccati equation in (Wang 1998, Theorem 1) as no tuning of parameters is required. Moreover, a state feedback law obtained in (Wang 1998, Theorem 1) is always a feasible solution of the LMI in Corollary 1, but not vice versa. Therefore, it can be seen that Corollary 1 recovers the result of (Wang 1998, Theorem 1). Similarly Corollary 2 recovers the result of (Park and Won 1999, Theorem 1).

3. Robust reliable control: sector nonlinearity case

3.1. *Problem Formulation*

This section considers the reliable control problem when the uncertain neutral system has sector-type non-linear actuators, that is,

$$\dot{x}(t)=(A+\Delta A(t))x(t)+(A_h+\Delta A_h(t))x(t-h) + (A_d+\Delta A_d(t))\,\dot{x}(t-d)+BS(u(t))+D\omega(t) \tag{32}$$

$$z(t)=Cx(t) \tag{33}$$

$$x(\theta)=\Phi(\theta), \qquad \forall\theta\in[-\tau,0], \tag{34}$$

where

$$u(t)=[u_1(t)\quad u_2(t)\quad \cdots \quad u_m(t)]\in\mathbb{R}^m$$

is the control input vector to the actuator, and

$$S(u(t))=[\sec_1(u_1(t))\quad \sec_2(u_2(t))\quad \cdots \quad \sec_m(u_m(t))]$$

is the control input vector to the plant with S composed of individual scalar sector non-linear functions $\sec_i(\cdot)$, $i=1,2,\ldots,m$, satisfying $\sec_i(0)=0$, for the ith input $u_i(t)$. It is assumed that the graphs of each $\sec_i(\cdot)$ considered lie inside the sector $[\sigma_{li},\sigma_{hi}]$ and mathematically, we have

$$\sigma_{li}u_i^2(t)\le u_i(t)\sec_i(u_i(t))\le\sigma_{hi}u_i^2(t),$$
$$0\le\sigma_{li}\le\sigma_{hi}<\infty.$$

Then the corresponding closed-loop system with the state feedback control law (5) is given by

$$\dot{x}(t)=(A_c+\Delta A(t))x(t)+(A_h+\Delta A_h(t))x(t-h) + (A_d+\Delta A_d(t))\,\dot{x}(t-d)+B\eta(t)+D\omega(t) \tag{35}$$

$$z(t)=Cx(t), \tag{36}$$

where

$$A_c=A+BM_aL$$
$$\eta(t)=S(Lx(t))-M_aLx(t)$$
$$M_a=\tfrac{1}{2}\mathrm{diag}(\sigma_{l1}+\sigma_{h1},\ldots,\sigma_{lm}+\sigma_{hm}).$$

Obviously, $\eta(t)$ satisfies the following inequality

$$\eta^{\mathrm{T}}(t)\eta(t)\le x^{\mathrm{T}}(t)L^{\mathrm{T}}M_s^2Lx(t)$$

where

$$M_s=\tfrac{1}{2}\mathrm{diag}(\sigma_{h1}-\sigma_{l1},\ldots,\sigma_{hm}-\sigma_{lm}).$$

The robust reliable control problem due to sector-type nonlinear actuators is stated in the following.

Problem RRCSN: To determine the state feedback law (5) such that the following requirements are met.

- Q1 closed-loop system from (35) to (36) is asymptotically stable.
- Q2 an H_∞ norm bound constraint on the disturbance attenuation is guaranteed. That is, under zero initial conditions

$$\|z(t)\|_2\le\gamma\|\omega(t)\|_2,$$

where $\gamma>0$ is a given constant.

3.2. *Main result*

This section begins by introducing two lemmas that will play important roles for the proof of main result here. The first lemma provides a sufficient condition to ensure the stabilization of neutral delay system from (32) to (34) without parameter uncertainties and disturbance input.

Lemma 4: *Consider the neutral delay system*

$$\dot{x}(t)=Ax(t)+A_hx(t-h)+A_d\dot{x}(t-d)+BS(u(t)) \tag{37}$$

$$z(t)=Cx(t) \tag{38}$$

$$x(\theta)=\Phi(\theta), \qquad \forall\theta\in[-\tau,0]. \tag{39}$$

If there exist matrices $P>0$, $Q>0$, $S>0$, $N>0$ *and* L *such that the following MIs hold*

$$\begin{bmatrix} U & V & PA_h & PB \\ V^{\mathrm{T}} & -R & 0 & 0 \\ A_h^{\mathrm{T}}P & 0 & -S & 0 \\ B^{\mathrm{T}}P & 0 & 0 & -I \end{bmatrix} < 0 \tag{40}$$

$$\begin{bmatrix} -N & L^{\mathrm{T}}M_s \\ M_sL & -I \end{bmatrix} \le 0, \tag{41}$$

where

$$\begin{aligned} U &= PA_c + A_c^{\mathrm{T}}P + S + Q + N \\ V &= (S + Q + N + PA_c)A_d \\ R &= Q - A_d^{\mathrm{T}}(S + Q + N)A_d \\ A_c &= A + BM_aL, \end{aligned} \tag{42}$$

then the state feedback control law (5) *stabilizes neutral delay system from* (37) *to* (39) *independently of the time delays h and d.*

Proof: With Lyapunov functional candidate (17), the time derivative of $V(x_t)$ along the trajectory of system from (35) and (36) when the uncertainties are zero and $\omega(t) = 0$ is given by

$$\begin{aligned} \dot{V}(x_t) &= 2(x(t) - A_dx(t-d))^{\mathrm{T}}P(A_cx(t) \\ &\quad + A_hx(t-h) + B\eta(t)) \\ &\quad + x^{\mathrm{T}}(t)(S+Q)x(t) - x^{\mathrm{T}}(t-d)Qx(t-d) \\ &\quad - x^{\mathrm{T}}(t-h)Sx(t-h) \\ &= (x(t) - A_dx(t-d))^{\mathrm{T}}(PA_c + A_c^{\mathrm{T}}P + S + Q + N) \\ &\quad \times (x(t) - A_dx(t-d)) - x^{\mathrm{T}}(t-d)Rx(t-d) \\ &\quad + 2(x(t) - A_dx(t-d))^{\mathrm{T}}(S + Q + N + PA_c) \\ &\quad \times A_dx(t-d) + 2(x(t) - A_dx(t-d))^{\mathrm{T}}PA_hx(t-h) \\ &\quad + 2(x(t) - A_dx(t-d))^{\mathrm{T}}PB\eta(t) \\ &\quad - x^{\mathrm{T}}(t-h)Sx(t-h) - x^{\mathrm{T}}(t)Nx(t). \end{aligned}$$

Noting the definition of the operator $\mathcal{D}$, the above equality can be rewritten as

$$\begin{aligned} \dot{V}(x_t) &= \mathcal{D}^{\mathrm{T}}(x(t))(PA_c + A_c^{\mathrm{T}}P + S + Q + N) \\ &\quad \times \mathcal{D}(x(t)) + 2\mathcal{D}^{\mathrm{T}}(x(t)) \\ &\quad \times (S + Q + N + PA_c)A_dx(t-d) \\ &\quad + 2\mathcal{D}^{\mathrm{T}}(x(t))PA_hx(t-h) + 2\mathcal{D}^{\mathrm{T}}(x(t)) \\ &\quad \times PB\eta(t) - x^{\mathrm{T}}(t-d)Rx(t-d) \\ &\quad - x^{\mathrm{T}}(t-h)Sx(t-h) - x^{\mathrm{T}}(t)Nx(t). \end{aligned}$$

By considering (42) and Lemma 1, it follows that

$$\begin{aligned} \dot{V}(x_t) &\le \mathcal{D}^{\mathrm{T}}(x(t))(PA_c + A_c^{\mathrm{T}}P + S + Q + N \\ &\quad + (S + Q + N + PA_c)A_dR^{-1}A_d^{\mathrm{T}} \\ &\quad \times (S + Q + N + PA_c)^{\mathrm{T}} \\ &\quad + PA_hS^{-1}A_h^{\mathrm{T}}P + PBB^{\mathrm{T}}P)\mathcal{D}(x(t)) + x^{\mathrm{T}}(t) \\ &\quad \times [L^{\mathrm{T}}M_s^2L - N]x(t). \end{aligned}$$

From (40) and (41), we can obtain that

$$\dot{V}(x_t) < 0,$$

i.e. the system from (37) to (39) is asymptotically stable. The proof is completed. ■

The second lemma provides a sufficient condition to ensure the stability of neutral system from (37) to (39) with disturbance attenuation γ.

Lemma 5: *Consider the neutral delay system*

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_hx(t-h) + A_d\dot{x}(t-d) \\ &\quad + BS(u(t)) + D\omega(t) \end{aligned} \tag{43}$$

$$z(t) = Cx(t) \tag{44}$$

$$x(\theta) = \Phi(\theta), \qquad \forall\theta \in [-\tau, 0]. \tag{45}$$

If there exist matrices $P > 0$, $Q > 0$, $S > 0$, $N > 0$, and L such that the following MIs hold

$$\begin{bmatrix} U & V & PA_h & PB & PD \\ V^{\mathrm{T}} & -R & 0 & 0 & 0 \\ A_h^{\mathrm{T}}P & 0 & -S & 0 & 0 \\ B^{\mathrm{T}}P & 0 & 0 & -I & 0 \\ D^{\mathrm{T}}P & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0 \tag{46}$$

$$\begin{bmatrix} -N & L^{\mathrm{T}}M_s & C^{\mathrm{T}} \\ M_sL & -I & 0 \\ C & 0 & -I \end{bmatrix} \le 0, \tag{47}$$

then the state feedback control law (5) *stabilizes neutral delay system from* (43) *to* (45) *with disturbance attenuation γ, independently of the time delays h and d.*

Proof: By Lemma 4 and inequality (46), the asymptotic stability of the closed-loop system from (35)–(36) when the uncertainties are zero and $\omega(t) = 0$ can be easily deduced.

Now, consider the disturbance attenuation problem. The time derivative of $V(x_t)$ along the trajectory of system from (35) and (36) when the uncertainties are zero is given by

$$\begin{aligned}\dot V(x_t) &= \mathcal{D}^{\mathrm T}(x(t))\{PA_c + A_c^{\mathrm T}P + S + Q + N\}\\ &\quad\times \mathcal{D}(x(t)) + 2\mathcal{D}^{\mathrm T}(x(t))(S + Q + N + PA_c)\\ &\quad\times A_d x(t-d)\\ &\quad + 2\mathcal{D}^{\mathrm T}(x(t))PA_h x(t-h) + 2\mathcal{D}^{\mathrm T}(x(t))\\ &\quad\times PB\eta(t) + 2\mathcal{D}^{\mathrm T}(x(t))PD\omega(t)\\ &\quad - x^{\mathrm T}(t-d)Rx(t-d)\\ &\quad - x^{\mathrm T}(t-h)Sx(t-h) - x^{\mathrm T}(t)Nx(t).\end{aligned}$$

By Lemma 1 and Hamiltonian (18), we have

$$\begin{aligned}H(x(t),\omega(t),t) &= \dot V(x_t) + z^{\mathrm T}(t)z(t) - \gamma^2\omega^{\mathrm T}(t)\omega(t)\\ &\le \mathcal{D}^{\mathrm T}(x(t))(PA_c + A_c^{\mathrm T}P + S + Q\\ &\quad + N + (S + Q + N + PA_c)\\ &\quad\times A_d R^{-1}A_d^{\mathrm T}(S + Q + N + PA_c)^{\mathrm T}\\ &\quad + PA_h S^{-1}A_h^{\mathrm T}P + PBB^{\mathrm T}P + \gamma^{-2}PDD^{\mathrm T}P)\\ &\quad\times \mathcal{D}(x(t)) + x^{\mathrm T}(t)\\ &\quad\times[(L^{\mathrm T}M_s^2L + C^{\mathrm T}C) - N]x(t).\end{aligned}$$

From (46) and (47), we can obtain that

$$H(x(t),\omega(t),t) < 0.$$

Thus the proof is completed. ■

The main result is given as follows.

Theorem 2: *If there exist scalars $\varepsilon_i > 0$, $i = 1,\ldots,5$, matrices $P > 0$, $Q > 0$, $S > 0$, $N > 0$, and L such that the following MIs hold.*

$$\begin{bmatrix} \bar U & 0 & PA_h & L_1 & 0 & PB & PD\\ 0 & -\bar R & 0 & 0 & L_2 & 0 & 0\\ A_h^{\mathrm T}P & 0 & -\bar S & 0 & 0 & 0 & 0\\ L_1^{\mathrm T} & 0 & 0 & -J_1 & 0 & 0 & 0\\ 0 & L_2^{\mathrm T} & 0 & 0 & -J_2 & 0 & 0\\ B^{\mathrm T}P & 0 & 0 & 0 & 0 & -I & 0\\ D^{\mathrm T}P & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I\end{bmatrix} < 0 \quad (48)$$

$$\begin{bmatrix} -N & L^{\mathrm T}M_s & C^{\mathrm T}\\ M_sL & -I & 0\\ C & 0 & -I\end{bmatrix} \le 0, \quad (49)$$

where

$$\bar U = U + \varepsilon_1 H_1^{\mathrm T}H_1,$$

$$\bar R = R - \varepsilon_5 H_3^{\mathrm T}H_3,$$

$$\bar S = S - \varepsilon_2 H_2^{\mathrm T}H_2,$$

$$L_1 = [PF \quad PF \quad PF \quad Q + S + N + PA_c],$$

$$L_2 = [H_3^{\mathrm T} \quad A_d^{\mathrm T} \quad A_d^{\mathrm T}(Q + S + N)F],$$

$J_1 = \mathrm{diag}[\varepsilon_1 I \quad \varepsilon_2 I \quad \varepsilon_3 I \quad I - \varepsilon_3 H_1^{\mathrm T}H_1]$ and

$$J_2 = \mathrm{diag}[\varepsilon_4 I \quad I - \varepsilon_4 FF^{\mathrm T} \quad \varepsilon_5 I - F^{\mathrm T}(Q + S + N)F].$$

then, with state feedback control law (5)*, the uncertain closed-loop system from* (35) *and* (36) *is robustly asymptotically stable with disturbance attenuation γ, independently of the time delays h and d.*

Proof: Under the conditions of Theorem 2, we notice that $\varepsilon_i > 0$, $i = 3, 4, 5$ satisfying

$$I - \varepsilon_3 H_1^{\mathrm T}H_1 > 0$$

$$I - \varepsilon_4 FF^{\mathrm T} > 0$$

$$\varepsilon_5 I - F^{\mathrm T}(Q + S + N)F > 0.$$

Then by Lemma 1, we have the following inequalities:

$$P\Delta A(t) + \Delta A^{\mathrm T}(t)P \le \varepsilon_1^{-1}PFF^{\mathrm T}P + \varepsilon_1 H_1^{\mathrm T}H_1 \quad (50)$$

$$\begin{bmatrix} 0 & P\Delta A_h(t)\\ \Delta A_h^{\mathrm T}(t)P & 0\end{bmatrix} \le \begin{bmatrix} \varepsilon_2^{-1}PFF^{\mathrm T}P & 0\\ 0 & \varepsilon_2 H_2^{\mathrm T}H_2\end{bmatrix} \quad (51)$$

$$\begin{bmatrix} 0 & (S + Q + N + PA_c + P\Delta A(t))(A_d + \Delta A_d(t))\\ (A_d + \Delta A_d(t))^{\mathrm T}(S + Q + N + PA_c + P\Delta A(t)^{\mathrm T} & 0\end{bmatrix}$$

$$\leq \begin{bmatrix} (S+Q+N+PA_c+P\Delta A(t))(S+Q+N+PA_c+P\Delta A(t))^{\mathrm{T}} & 0 \\ 0 & (A_d+\Delta A_d(t))^{\mathrm{T}}(A_d+\Delta A_d(t)) \end{bmatrix}$$

$$\leq \begin{bmatrix} Q_1(I-\varepsilon_3 H_1^{\mathrm{T}} H_1)^{-1} Q_1^{\mathrm{T}} + \varepsilon_3^{-1} PFF^{\mathrm{T}} P & 0 \\ 0 & A_d^{\mathrm{T}}(I-\varepsilon_4 FF^{\mathrm{T}})^{-1} A_d + \varepsilon_4^{-1} H_3^{\mathrm{T}} H_3 \end{bmatrix}, \quad (52)$$

where

$$Q_1 = Q + S + N + PA_c$$

and

$$\begin{aligned}(A_d+\Delta A_d(t))^{\mathrm{T}}&(Q+S+N)(A_d+\Delta A_d(t)) \\ &\leq A_d^{\mathrm{T}}(Q+S+N)A_d + \varepsilon_5 H_3^{\mathrm{T}} H_3 \\ &\quad + A_d^{\mathrm{T}}(Q+S+N)F\big[\varepsilon_5 I - F^{\mathrm{T}}(Q+S+N)F\big]^{-1} \\ &\quad \times F^{\mathrm{T}}(Q+S+N)A_d \end{aligned} \quad (53)$$

By Schur complements, it follows from (48) and the inequalities (50)–(53) that

$$\begin{bmatrix} \hat{U} & \hat{V} & P(A_h+\Delta A_h(t))^{\mathrm{T}} & PB & PD \\ \hat{V}^{\mathrm{T}} & -\hat{R} & 0 & 0 & 0 \\ (A_h+\Delta A_h(t))P & 0 & -S & 0 & 0 \\ B^{\mathrm{T}}P & 0 & 0 & -I & 0 \\ D^{\mathrm{T}}P & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} < 0,$$

where

$$\hat{U} = U + P\Delta A(t) + \Delta A^{\mathrm{T}}(t)P,$$

$$\hat{V} = (S+Q+N+PA_c+P\Delta A(t))(A_d+\Delta A_d(t)) \text{ and}$$

$$\hat{R} = Q - (A_d+\Delta A_d(t))^{\mathrm{T}}(S+Q+N)(A_d+\Delta A_d(t)).$$

Thus, the closed-loop system in (35) and (36) is robustly stable with disturbance attenuation γ according to Lemma 5. ■

To design a controller based on Theorem 2, an iterative LMI solution process summarized in the following schematic algorithm is proposed. Let $(48)'$ and $(49)'$ denote MIs (48) and (49) with

$$M_a = \tfrac{1}{2}\mathrm{diag}(\hat{\sigma}_{l1}+\hat{\sigma}_{h1},\ldots,\hat{\sigma}_{lm}+\hat{\sigma}_{hm})$$

$$M_s = \tfrac{1}{2}\mathrm{diag}(\hat{\sigma}_{h1}-\hat{\sigma}_{l1},\ldots,\hat{\sigma}_{hm}-\hat{\sigma}_{lm}).$$

Algorithm RRCSN: Given system parameters A, A_h, A_d, B, C, D, F, H_1, H_2, H_3, and design specifications γ, σ_{hi}, and σ_{li}, for $i=1,2,\ldots,m$.

Step 1. Find a feasible solution L of LMI (20) with $B_{\bar{\Omega}} = B$ (B_{Ω} does not appear). If this is not feasible, GOTO Step 4.

Step 2. Let $\mu_{hi} \geq 0$ and $\mu_{li} \geq 0$ for $i=1,2,\ldots,m$, be adjustable parameters, and let $\hat{\sigma}_{li} = \hat{\sigma}_{hi} = 1$ for $i=1,2,\ldots,m$.
- (a) Set $i=1$.
- (b) Substitute L into $(48)'$ and $(49)'$ and find a feasible solution set of variables P, Q, S, N, ε_1, ε_2, ε_3, ε_4 and ε_5.
- (c) With P, Q, S, N, ε_1, ε_2, ε_3, ε_4, ε_5, let
$$\hat{\sigma}_{hi} = \hat{\sigma}_{hi} + \mu_{hi}, \quad \hat{\sigma}_{li} = \hat{\sigma}_{li} - \mu_{li}.$$
For any i, if $\hat{\sigma}_{hi} > \sigma_{hi}$, then set $\hat{\sigma}_{hi} = \sigma_{hi}$; if $\hat{\sigma}_{li} < \sigma_{li}$, then set $\hat{\sigma}_{li} = \sigma_{li}$. Solve LMIs $(48)'$ and $(49)'$ for a feasible solution L. If a feasible L cannot be found, GOTO Step 4.
- (d) Set $i = i+1$. If $i \leq m$, GOTO Step 2(b).

Step 3. If $\hat{\sigma}_{hi} = \sigma_{hi}$ and $\hat{\sigma}_{li} = \sigma_{li}$ for $i=1,2,\ldots,m$, then a desired state feedback controller L is found. STOP.

Step 4. The algorithm cannot found a state feedback controller satisfying the given design specifications.

Remark 2: Step 1 corresponds to finding a feasible controller assuming there is no sector-type actuator non-linearity. If even under this condition, a feasible L cannot be found, then it is likely that a solution of the problem does not exist. Note also that the feasibility problem in Step 2(b) involves LMIs.

Remark 3: In the case when $\sigma_{hi} = \mu_i$ and $\sigma_{li} = 1/\mu_i$, where $\mu_i > 1$, for $i=1,2,\ldots,m$, we can find the largest nonlinear sector, in an angular sense symmetric measured about the unity slope, through determining the maximum allowable $\bar{\mu}_i$ for μ_i, which ensures that the uncertain closed-loop system from (35) and (36) robustly asymptotically stable with disturbance attenuation γ for any $1 < \mu_i < \bar{\mu}_i$, independently of the time delays h and d. The maximum allowable $\bar{\mu}_i$ can be determined by using Algorithm RRCSN.

4. Numerical examples

Here, two numerical examples are given to demonstrate the applicability of the proposed approach.

Example 1: Consider the uncertain neutral delay system from (1) to (3) with parameters as follows:

$$A = \begin{bmatrix} 0.2 & 1 & 0 \\ -2 & 0.6 & 0 \\ 0 & 0 & -5 \end{bmatrix}, \quad A_h = \begin{bmatrix} 0.3 & 0.1 & 0.2 \\ 0.2 & 0 & 0 \\ 0 & -0.3 & 0 \end{bmatrix},$$

$$A_d = \begin{bmatrix} -0.1 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0.01 & 0.02 \end{bmatrix},$$

$$B = \begin{bmatrix} -9.1 & 1 & 0.3 \\ 0 & 0.6 & 0 \\ 0.8 & 0 & -1.5 \end{bmatrix},$$

$$B_{\bar{\Omega}} = \begin{bmatrix} -9.1 & 1 \\ 0 & 0.6 \\ 0.8 & 0 \end{bmatrix}, \quad B_{\Omega} = \begin{bmatrix} 0.3 \\ 0 \\ -1.5 \end{bmatrix}$$

$$F = \begin{bmatrix} 0.1 \\ 0 \\ -0.2 \end{bmatrix}, \quad H_1 = [0 \quad 0.05 \quad 0.01],$$

$$H_2 = [0.1 \quad 0.2 \quad 0.01], \quad H_3 = [0.1 \quad 0 \quad 0.01],$$

$$C = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 & 0 & 0.3 \\ 0.2 & 0.2 & 0 \\ 0 & 0 & -0.5 \end{bmatrix}.$$

The matrix A has eigenvalues $0.4 \pm 1.4j$ and -5. It is required to construct the state feedback controller with form (21) that can robustly stabilize the uncertain neutral delay system from (1) to (3) with disturbance attenuation γ, independently of the time delays h and d, for all admissible uncertainties as well as all actuator faults corresponding to $\varphi \in \Omega$. If the H_∞-norm bound is prespecified as $\gamma = 1.3$, then, by using Matlab LMI Control Toolbox and solving the LMI (20), we obtain

$$X = \begin{bmatrix} 2.5044 & 1.3718 & -0.4953 \\ 1.3718 & 1.1490 & -0.5260 \\ -0.4953 & -0.5260 & 101.6707 \end{bmatrix},$$

$$Q = \begin{bmatrix} 2.9411 & -0.3979 & 0.9242 \\ -0.3979 & 0.2402 & -1.2010 \\ 0.9242 & -1.2010 & 251.8763 \end{bmatrix},$$

$$S = \begin{bmatrix} 4.6003 & -0.3626 & -0.4015 \\ -0.3626 & 1.2673 & -1.2586 \\ -0.4015 & -1.2586 & 253.6773 \end{bmatrix},$$

$$\epsilon_1 = 1.2172 \times 10^3, \quad \epsilon_2 = 0.1515 \times 10^3,$$

$$\epsilon_3 = 0.0106 \times 10^3, \quad \epsilon_4 = 0.0601 \times 10^3,$$

$$\epsilon_5 = 0.0858 \times 10^3.$$

Hence, from Theorem 1, a desired state feedback controller can be chosen as

$$u(t) = \begin{bmatrix} 10.5042 & -12.5505 & -0.0216 \\ -0.3270 & -0.1328 & -0.0023 \\ -0.3485 & 0.4231 & 0.0152 \end{bmatrix} x(t).$$

Example 2: Consider the uncertain neutral delay system from (32) to (34) with the same system parameters as in Example 1 except $B = B_{\bar{\Omega}}$. This can be viewed as having the third actuator becomes disconnected from the input in Example 1. On the other hand, the inputs now suffer from sector-type non-linearity with $S(u(t)) = [\sec_1(u_1(t)) \quad \sec_2(u_2(t))]$ where

$$0.65u_1^2 \le u_1 \sec_1(u_1) \le 3.50u_1^2,$$

$$0.74u_2^2 \le u_2 \sec_2(u_2) \le 2.50u_2^2. \tag{54}$$

It is required to construct the state feedback controller with form (5) that can robustly stabilize the uncertain neutral delay system from (32) to (34) with disturbance attenuation γ, independently of the time delays h and d, for all admissible uncertainties. By using Matlab LMI Control Toolbox and Algorithm RRCSN, a desired state feedback controller satisfying the sector-type specification in (54) and $\gamma = 1.3$ can be found as

$$u(t) = \begin{bmatrix} 2.8287 & -0.3164 & 0.5289 \\ -1.5521 & -2.1990 & -1.5833 \end{bmatrix} x(t)$$

from Theorem 2. Also, we have

$$P = \begin{bmatrix} 0.4476 & -0.1071 & 0.5339 \\ -0.1071 & 2.1154 & 0.0612 \\ 0.5339 & 0.0612 & 4.9331 \end{bmatrix},$$

$$Q = \begin{bmatrix} 1.1050 & 0.0731 & 0.7550 \\ 0.0731 & 0.0475 & -0.0960 \\ 0.7550 & -0.0960 & 8.3665 \end{bmatrix},$$

$$S = \begin{bmatrix} 2.2775 & 0.1891 & 0.5497 \\ 0.1891 & 0.1606 & -0.1538 \\ 0.5497 & -0.1538 & 8.3010 \end{bmatrix},$$

$$N = \begin{bmatrix} 18.5219 & 0.8703 & 5.5840 \\ 0.8703 & 3.9702 & 2.2346 \\ 5.5840 & 2.2346 & 10.8397 \end{bmatrix},$$

$$\epsilon_1 = 6.1267, \quad \epsilon_2 = 0.5851, \quad \epsilon_3 = 46.7667,$$

$$\epsilon_4 = 13.2790, \quad \epsilon_5 = 33.3421.$$

5. Conclusion

This paper dealt with robust reliable control problems for a class of uncertain neutral delay systems. Two cases of actuator imperfections were considered. The first case corresponded to actuator faults among a prespecified subset of actuators, the other focused on sector-type actuator non-linearity. Attention was paid to the design of state feedback controllers that guaranteed, for all admissible uncertainties as well as actuator faults or sector-type actuator non-linearity, that the closed-loop system was robustly asymptotically stable with H_∞ disturbance attenuation, independently of the time delays. The resulting neutral delay control system provided guaranteed robust reliable stability in spite of possible actuator faults or sector-type actuator non-linearity.

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