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Comments on 'Design of Decentralized Control for Symmetrically Interconnected Systems'*

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Key Words—Large-scale systems; decentralized control; pole assignment.

Abstract—It is shown in this note that the Lemma in Kashlan and Geneidy (1996) is not correct. Moreover, two kinds of decentralized pole assignment problems for symmetrically interconnected systems are studied and lower-dimensional necessary and sufficient conditions for the problems to be solvable are presented. © 1998 Elsevier Science Ltd. All rights reserved.

1. *Counter examples*

Recently, Kashlan and Geneidy (1996) studied the design of decentralized control for symmetrically interconnected systems. They claimed that the Lemma in Kashlan and Geneidy (1996) presented a necessary and sufficient condition for the decentralized control to exist. In this section, two examples are given to shown that the condition given in the Lemma is neither necessary nor sufficient.

The system considered in Kashlan and Geneidy (1996) composed of N symmetrically interconnected linear controllable subsystems, the ith subsystem is given by

$$\dot{x}_i = A_i x_i + \sum_{j=1, j\neq i}^{N} A_{12} x_j + \sum_{j=1}^{N} B_{ij} u_j, \quad i = 1, \ldots, N, \qquad (1)$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^{m_i}$, $(i = 1, \ldots, N)$ are the local state and control input vectors, respectively. A_i $A_{12} \in \mathbb{R}^{n\times n}$, $B_{ij} \in \mathbb{R}^{n\times m_j}$.

The problem studied in Kashlan and Geneidy (1996) is finding a decentralized controller

$$u_i = K_i x_i, \quad K_i \in \mathbb{R}^{m_i \times n}, \quad i = 1, \ldots, N, \qquad (2)$$

to assign a prescribed spectrum Ω for the closed-loop composite system, where $\Omega = \{\lambda_i, i = 1, \ldots, Nn\}$.

Remark 1. Since the interconnection matrices of the system (1) are all A_{12}, the dimensions of x_i $(i = 1, \ldots, N)$ should be the same, that is $n_1 = n_2 = \cdots = n_N$ in Kashlan and Geneidy (1996).

The following Lemma was introduced by Kashlan and Geneidy (1996).

Lemma. A class of decentralized controls of the form (2) exists if and only if the open-loop system and the closed-loop system have no eigenvalue in common, that is

$$\sigma(A) \cap \Omega = \Phi.$$

where $\sigma(A)$ denotes the spectrum of A and

$$A = \begin{bmatrix} A_1 & A_{12} & \cdots & A_{12} \\ A_{12} & A_2 & \cdots & A_{12} \\ \vdots & \vdots & & \vdots \\ A_{12} & A_{12} & \cdots & A_N \end{bmatrix}.$$

In fact, the condition given in the above lemma is neither necessary nor sufficient. We give two simple examples to show this.

Example 1. Consider the system

$$\dot{x}_1 = A_1 x_1 + A_{12} x_2 + B_{11} u_1 + B_{12} u_2,$$
$$\dot{x}_2 = A_2 x_2 + A_{12} x_1 + B_{21} u_1 + B_{22} u_2, \qquad (3)$$

where

$$A_1 = \begin{bmatrix} 0 & 2 \\ -1 & -3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 4 \\ 0 & -2 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},$$
$$B_{11} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Choose $\Omega = \{-2, -4, -2+\sqrt{2}, -2-\sqrt{2}\}$. Since $\sigma(A) = \{0, -1, -2, -2\}$, hence $\sigma(A) \cap \Omega = \{-2\} \neq \Phi$. But it is easy to testify that when applying the decentralized controller

$$u_1 = K_1 x_1, \qquad u_2 = K_2 x_2. \qquad (4)$$

to the system (3), where $K_1 = [0, -1]$, $K_2 = [-1, -2]$, the spectrum of the closed-loop system is Ω.

Example 2. Consider the system

$$\dot{x}_1 = A_1 x_1 + A_{12} x_2 + B_{11} u_1 + B_{12} u_2,$$
$$\dot{x}_2 = A_2 x_2 + A_{12} x_1 + B_{21} u_1 + B_{22} u_2, \qquad (5)$$

where

$$A_1 = A_2 = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},$$
$$B_{11} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

It is easy to see that $\sigma(A) = \{0, -2, -1+j, -1-j\}$ $(j = \sqrt{-1})$ and the pairs (A_1, B_{11}) and (A_2, B_{22}) are all controllable but $\lambda_1 = -1+j$ and $\lambda_2 = -1-j$ are decentralized fixed modes of the system (5). So there does not exist a decentralized controller of the form (4) such that the spectrum of the closed-loop system is $\Omega = \{-1, -1, -1, -1\}$. Though $\sigma(A) \cap \Omega = \Phi$. (Noting that there do exist decentralized controllers of the form (4) such that the closed-loop system is asymptotically stable. For example, if we choose $K_1 = [-1, -1]$, $K_2 = [-2, -2]$, then the spectrum of the closed-loop system is $\Omega^1 = \{-6.1926, -0.8074, -1+j, -1-j\} \subset C^-$.)

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Remark 2. Example 1 shows that the condition $\sigma(A) \cap \Omega = \Phi$ is not necessary for the decentralized controls to exist. Example 2 shows that the condition is not sufficient, either. Moreover, the design procedure developed in Kashlan and Geneidy (1996) also needs to be improved. In our opinion, at least the following two questions should be clearly answered:

(1) Under what condition does there exist a θ^* such that $W(\theta^*)V^{-1}(\theta^*) = \text{diag}[K_1, \ldots, K_N]$? Since the Lemma given in Kashlan and Geneidy (1996) is incorrect, the condition should be reconsidered.
(2) How to choose the θ^* such that $W(\theta^*)V^{-1}(\theta^*) = \text{diag}[K_1, \ldots, K_N]$ and $V^{-1}(\theta^*) = V^{\mathrm{T}}(\theta^*)$? Kashlan and Geneidy (1996) claimed that an iterative algorithm was applied. What is the algorithm? How to guarantee the convergence of the algorithm?

2. *Decentralized pole assignment*

In this section, we study the decentralized pole assignment problem for the "symmetrically interconnected systems". Such systems were considered in Sundareshan and Elbanna (1991). The system composed of N subsystems, the ith subsystem is given by

$$\dot{x}_i = A_1 x_i + \sum_{j=1, j \neq i}^{N} A_{12} x_j + B_1 u_i, \tag{6}$$

where $i = 1, 2, \ldots, N$, and $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$, $(i = 1, \ldots, N)$ are the n-, m-dimensional local state and control input vectors, respectively. A_1 $A_{12} \in \mathbb{R}^{n \times n}$, $B_1 \in \mathbb{R}^{n \times m}$. Then the state-space model of the overall system is

$$\dot{x} = Ax + Bu, \tag{7}$$

where $x = (x_1^{\mathrm{T}}, \ldots, x_N^{\mathrm{T}})^{\mathrm{T}}$, $u = (u_1^{\mathrm{T}}, \ldots, u_N^{\mathrm{T}})^{\mathrm{T}}$. And matrices $A \in \mathbb{R}^{Nn \times Nn}$, $B \in \mathbb{R}^{Nn \times Nm}$ have the structure

$$A = \begin{bmatrix} A_1 & A_{12} & \cdots & A_{12} \\ A_{12} & A_1 & \cdots & A_{12} \\ \vdots & \vdots & & \vdots \\ A_{12} & A_{12} & \cdots & A_1 \end{bmatrix}, \tag{8}$$

$$B = \text{diag}[B_1, \ldots, B_1].$$

Remark 3. "Symmetrically interconnected systems" are encountered in electric power systems, industrial manipulators, computer networks, etc. (Lunze, 1986; Sundareshan and Elbanna, 1991). These kind of systems were called "symmetric composite systems" by Lunze (1986). Because of the special structure of the system, many analyses and design problems for it can be simplified (Lunze, 1986; Sundareshan and Elbanna, 1991; Liu, 1992; etc.).

This section considers the following problem for the system (7).

Decentralized pole assignment problem:

Let U be any nonempty symmetric open subset of the complex plane (i.e., if $\lambda \in U$, then its complex conjugate $\lambda^* \in U$), then the decentralized pole assignment problem for the system (7) is finding a decentralized controller such that all poles of the resulting closed-loop system are contained in U.

For the system (7), let us denote

$$A_d = A_1 - A_{12}, \quad A_o = A_1 + (N-1)A_{12}. \tag{9}$$

Sundareshan and Elbanna (1991) studied one kind of decentralized pole assignment problem for the system (7) and got the following result.

Theorem 1 (Sundareshan and Elbanna, 1991). There exists a solution to the decentralized pole assignment problem for the system (7) by the decentralized controller of the form

$$u_i = K_0 x_i, \quad i = 1, \ldots, N, \ K_0 \in \mathbb{R}^{m \times n}, \tag{10}$$

if and only if

$$\sigma(A_d + B_1 K_0) \subset U, \qquad \sigma(A_o + B_1 K_0) \subset U. \tag{11}$$

Since all the subsystems in the system (7) are identical, it is an intuitive idea to use decentralized controller of the form (10). But decentralized controller of the form (10) cannot be optimal for all cases for the system (7) (see Example 3 in this section). Hence, we consider more general decentralized controller for the system (7), we consider the case when part of the subsystems have the same gain matrices $K_1 \in \mathbb{R}^{m \times n}$, and the other part of the subsystems have the same gain matrices $K_2 \in \mathbb{R}^{m \times n}$.

The main results of this section are the following two theorems, the proofs are in the appendix.

Theorem 2. There exists a solution to the decentralized pole assignment problem for the system (7) by the decentralized controller of the form

$$u_1 = K_1 x_1, \quad u_i = K_2 x_i \quad (i = 2, \ldots, N), \ K_1, K_2 \in \mathbb{R}^{m \times n}, \tag{12}$$

if and only if

$$\sigma(A_d + B_1 K_2) \subset U \tag{13}$$

and

$$\sigma\left(\begin{bmatrix} A_1 + B_1 K_1 & \sqrt{N-1}A_{12} \\ \sqrt{N-1}A_{12} & A_1 + (N-2)A_{12} + B_1 K_2 \end{bmatrix}\right) \subset U. \tag{14}$$

Theorem 3. For positive integer l such that $2 \leq l \leq N-2$, there exists a solution to the decentralized pole assignment problem for the system (7) by the decentralized controller of the from

$$u_i = K_1 x_i \quad (i = 1, \ldots, l),$$
$$u_i = K_2 x_i \quad (i = l+1, \ldots, N), \ K_1, K_2 \in \mathbb{R}^{m \times n}, \tag{15}$$

if and only if

$$\sigma(A_d + B_1 K_1) \subset U, \sigma(A_d + B_1 K_1) \subset U \tag{16}$$

and

$$\sigma\left(\begin{bmatrix} A_1 + (l-1)A_{12} + B_1 K_1 & \sqrt{l(N-l)}A_{12} \\ \sqrt{l(N-l)}A_{12} & A_1 + (N-l-1)A_{12} + B_1 K_2 \end{bmatrix}\right) \subset U. \tag{17}$$

From Theorem 2, for a given nonempty symmetric open subset U, in order to design a decentralized controller of the form (12), the following design procedure could be used.

Design procedure 1: (1) Select $K_2 \in \mathbb{R}^{m \times n}$, such that $\sigma(A_d + B_1 K_2) \subset U$. (2) Choose $K_1 \in \mathbb{R}^{m \times n}$, such that

$$\sigma\left(\begin{bmatrix} A_1 + B_1 K_1 & \sqrt{N-1}A_{12} \\ \sqrt{N-1}A_{12} & A_1 + (N-2)A_{12} + B_1 K_2 \end{bmatrix}\right) \subset U.$$

From Theorem 3, for a given nonempty symmetric open subset U, in order to design a decentralized controller of the form (15), the following design procedure could be used.

Design procedure 2:

Step 1. Select $K_1 \in \mathbb{R}^{m \times n}$ and $K_2 \in \mathbb{R}^{m \times n}$, such that $\sigma(A_d + B_1 K_1) \subset U$ and $\sigma(A_d + B_1 K_2) \subset U$.

Step 2. Let $l = 2$.

Step 3. Compute

$$\sigma\left(\begin{bmatrix} A_1 + (l-1)A_{12} + B_1 K_1 & \sqrt{l(N-l)}A_{12} \\ \sqrt{l(N-l)}A_{12} & A_1 + (N-l-1)A_{12} + B_1 K_2 \end{bmatrix}\right).$$

Step 4. Check the condition (17), if it is satisfied, then stop, otherwise, let $l = l + 1$.

Step 5. If $l \leq N-2$, then go back to step 3, otherwise, go back to step 1, select K_1 and K_2 again.

The following example shows that sometimes the decentralized pole assignment problem for the system (7) can be solved by the decentralized controller of the form (12) when it cannot be solved by the decentralized controller of the form (10).

Example 3. Consider the system given by equation (7) where $N = 10$ and

$$A_1 = \begin{bmatrix} 0 & 8.5 \\ 0 & 0 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

computing directly, we have

$$A_d = \begin{bmatrix} 0 & 9.5 \\ 0 & 0 \end{bmatrix}, \qquad A_o = \begin{bmatrix} 0 & -0.5 \\ 0 & 0 \end{bmatrix}.$$

Choose $U = C^-$, it is easy to see that for $K_0 = [k_{01}, k_{02}]$, $\sigma(A_d + B_1K_0) \subset C^-$ if and only if $k_{01} < 0$ and $k_{02} < 0$. $\sigma(A_0 + B_1K_0) \subset C^-$ if and only if $k_{01} > 0$ and $k_{02} < 0$. Hence there does not exist a K_0 such that equation (11) holds. From Theorem 1, the system (7) cannot be stabilized by the decentralized controller of the form (10). But if we choose $K_1 = [0.3, -10]$, $K_2 = [-10, 1]$, then equations (13) and (14) hold. From Theorem 2, the decentralized controller (12) stabilizes the system (7).

Remark 4. The more general form of decentralized controller

$$u_i = K_1x_i \quad (i = 1, \dots, l_1), \qquad u_i = K_2x_i \quad (i = l_1 + 1, \dots, l_2),$$

$$u_i = K_3x_i \quad (i = l_2 + 1, \dots, N). \tag{18}$$

(where l_1 and l_2 are positive integers such that $1 \le l_1 < l_2 \le N - 1$) for the system (7) can be studied by using the same method as those of Theorems 2 and 3. In this case, a $3n$-dimensional necessary and sufficient condition could be obtained. Generally speaking, the more general the decentralized controller is, the higher the dimension of the necessary and sufficient condition will be.

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References

Kashlan, A. El and M. El Geneidy (1996). Design of decentralized control for symmetrically interconnected systems. *Automatica*, **32**(3), 475–476.

Liu Xiaoping (1992). Output regulation of strongly coupled symmetrically interconnected systems. *Automatica*, **28**(5), 1037–1041.

Lunze J. (1986). Dynamics of strongly coupled symmetric composite systems. *Int. J. Control*, **44**(6), 1617–1640.

Sundareshan, M. K. and R. M. Elbanna (1991). Qualitative analysis and decentralized controllers synthesis for a class of large-scale systems with symmetrically interconnected subsystems. *Automatica*, **27**, 383–388.

Appendix—Proofs

For a positive integer p, we denote

$$S_p = \frac{1}{\sqrt{p}} \begin{bmatrix} 1 & 0 & \cdots & 0 & 1 \\ 0 & 1 & \cdots & 0 & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 1 \\ -1 & -1 & \cdots & -1 & 1 \end{bmatrix} \in \mathbb{R}^{p \times p}.$$

Then we have the following lemma.

Lemma 1. For positive integers p and q, let

$$D_p = \begin{bmatrix} a & b & \cdots & b \\ b & a & \cdots & b \\ \vdots & \vdots & & \vdots \\ b & b & \cdots & a \end{bmatrix} \in \mathbb{R}^{p \times p}, \quad E_{pq} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{p \times q}.$$

where a, b are two arbitrarily given numbers. Then the following two equality hold

$$S_p^{-1} D_p S_p = \begin{bmatrix} a - b & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & a - b & 0 \\ 0 & \cdots & 0 & a + (p-1)b \end{bmatrix} \in \mathbb{R}^{p \times p},$$

$$S_p^{-1} E_{pq} S_q = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & \sqrt{pq} \end{bmatrix} \in \mathbb{R}^{p \times q}.$$

Proof. By computing directly, we have

$$S_p^{-1} = \frac{1}{\sqrt{p}} \begin{bmatrix} p-1 & -1 & \cdots & -1 & -1 \\ -1 & p-1 & \cdots & -1 & -1 \\ \vdots & \vdots & & \vdots & \vdots \\ -1 & -1 & \cdots & p-1 & -1 \\ 1 & 1 & \cdots & 1 & 1 \end{bmatrix} \in \mathbb{R}^{p \times p}$$

and Lemma 1 follows immediately.

In order to prove Theorems 2 and 3, we further denote

$$T_{pi} = S_p \otimes I_i$$

where I_i is an $i \times i$ identity matrix and $\otimes$ denotes the Kronecker product.

Proof of Theorem 2. When applying the decentralized controller (12) to the system (7), the closed-loop system matrix becomes

$$A_{c1} = \begin{bmatrix} A_1 + B_1K_1 & A_{12} & A_{12} & \cdots & A_{12} \\ A_{12} & A_1 + B_1K_2 & A_{12} & \cdots & A_{12} \\ A_{12} & A_{12} & A_1 + B_1K_2 & \cdots & A_{12} \\ \vdots & \vdots & \vdots & & \vdots \\ A_{12} & A_{12} & A_{12} & \cdots & A_1 + B_1K_2 \end{bmatrix}. \tag{19}$$

From Lemma 1, we have

$$\sigma(A_{c1}) = \sigma\left(\begin{bmatrix} T_{1n}^{-1} & 0 \\ 0 & T_{(N-1)n}^{-1} \end{bmatrix} A_{c1} \begin{bmatrix} T_{1n} & 0 \\ 0 & T_{(N-1)n} \end{bmatrix}\right)$$

$$= \sigma\left(\begin{bmatrix} A_1 + B_1K_1 & & & & \sqrt{N-1}A_{12} \\ & A_d + B_1K_2 & & & \\ & & \ddots & & \\ & & & A_d + B_1K_2 & \\ \sqrt{N-1}A_{12} & & & & A_1 + (N-2)A_{12} + B_1K_2 \end{bmatrix}\right)$$

$$= \sigma\left(\begin{bmatrix} A_1 + B_1K_1 & \sqrt{N-1}A_{12} & & & \\ \sqrt{N-1}A_{12} & A_1 + (N-2)A_{12} + B_1K_2 & & & \\ & & A_d + B_1K_2 & & \\ & & & \ddots & \\ & & & & A_d + B_1K_2 \end{bmatrix}\right)$$

$$= \sigma\left(\begin{bmatrix} A_1 + B_1K_1 & \sqrt{N-1}A_{12} \\ \sqrt{N-1}A_{12} & A_1 + (N-2)A_{12} + B_1K_2 \end{bmatrix}\right) \cup \left(\bigcup^{N-2} \sigma(A_d + B_1K_2)\right)$$

Thus $\sigma(A_{c1}) \subset U$ if and only if $\sigma(A_d + B_1K_2) \subset U$ and

$$\sigma\left(\begin{bmatrix} A_1 + B_1K_1 & \sqrt{N-1}A_{12} \\ \sqrt{N-1}A_{12} & A_1 + (N-2)A_{12} + B_1K_2 \end{bmatrix}\right) \subset U .$$

Proof of the Theorem 3. When applying the decentralized controller (15) to the system (7), the closed-loop system matrix becomes

$$A_{cl} = \begin{bmatrix} A_1 + B_1K_1 & A_{12} & \cdots & A_{12} & A_{12} & A_{12} & \cdots & A_{12} \\ A_{12} & A_1 + B_1K_1 & \cdots & A_{12} & A_{12} & A_{12} & \cdots & A_{12} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ A_{12} & A_{12} & \cdots & A_1 + B_1K_1 & A_{12} & A_{12} & \cdots & A_{12} \\ A_{12} & A_{12} & \cdots & A_{12} & A_1 + B_1K_2 & A_{12} & \cdots & A_{12} \\ A_{12} & A_{12} & \cdots & A_{12} & A_{12} & A_1 + B_1K_2 & \cdots & A_{12} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ A_{12} & A_{12} & \cdots & A_{12} & A_{12} & A_{12} & \cdots & A_1 + B_1K_2 \end{bmatrix} \qquad (20)$$

From Lemma 1, we have

$$\sigma(A_{c1}) = \sigma\left(\begin{bmatrix} T_{ln}^{-1} & 0 \\ 0 & T_{(N-l)n}^{-1} \end{bmatrix} A_{c1} \begin{bmatrix} T_{ln} & 0 \\ 0 & T_{(N-l)n} \end{bmatrix}\right)$$

$$= \sigma\left(\begin{bmatrix} A_d + B_1K_1 & & & & & & \\ & \ddots & & & & & \\ & & A_d + B_1K_1 & & & & \\ & & & A_1 + (l-1)A_{12} + B_1K_1 & & & \sqrt{l(N-l)}A_{12} \\ & & & & A_d + B_1K_2 & & \\ & & & & & \ddots & \\ & & & & & & A_d + B_1K_2 & \\ & & & \sqrt{l(N-l)}A_{12} & & & & A_1 + (N-l-1)A_{12} + B_1K_{12} \end{bmatrix}\right)$$

$$= \sigma\left(\begin{bmatrix} A_1 + (l-1)A_{12} + B_1K_1 & \sqrt{l(N-l)}A_{12} & & & & & & \\ \sqrt{l(N-l)}A_{12} & A_1 + (N-l-1)A_{12} + B_1K_2 & & & & & & \\ & & A_d + B_1K_1 & & & & & \\ & & & \ddots & & & & \\ & & & & A_d + B_1K_1 & & & \\ & & & & & A_d + B_1K_2 & & \\ & & & & & & \ddots & \\ & & & & & & & A_d + B_1K_2 \end{bmatrix}\right)$$

$$= \sigma\left(\begin{bmatrix} A_1 + (l-1)A_{12} + B_1K_1 & \sqrt{l(N-l)}A_{12} \\ \sqrt{l(N-l)}A_{12} & A_1 + (N-l-1)A_{12} + B_1K_2 \end{bmatrix}\right) \cup \left(\bigcup^{l-1} \sigma(A_d + B_1K_1)\right) \cup \left(\bigcup^{N-l-1} \sigma(A_d + B_1K_2)\right).$$

Thus $\sigma(A_{cl}) \subset U$ if and only if $\sigma(A_d + B_1K_1) \subset U$, $\sigma(A_d + B_1K_2) \subset U$ and

$$\sigma\left(\begin{bmatrix} A_1 + (l-1)A_{12} + B_1K_1 & \sqrt{l(N-l)}A_{12} \\ \sqrt{l(N-l)}A_{12} & A_1 + (N-l-1)A_{12} + B_1K_2 \end{bmatrix}\right) \subset U$$

Authors reply

Paper [1] consists of two parts, we present here our reply only to the first part:

We agree with the authors for their remark (1), but it does not play any role in the proof of our lemma.

Example 1 does not apply to our paper [2], we refer to the introduction "The present paper considers a more general of large-scale systems, where a number of subsystems are interconnected by symmetrical interconnection through their states and *general interactions through their controls*". The counter example 1 ignores the control interactions.

As for Example 2 since the composite system is uncontrollable (containing 0 as fixed mode) it is meaningless to talk about eigenvalue assignment. But to be on alert one should bear in mind to test controllability of $[(A + \bar{A}), (B + \bar{B})]$ to ensure that the pair $(A + \bar{A})$ and $(B + \bar{B})$ has no fixed mode(s), i.e. $\bigcap \sigma[A + \bar{A}) + (B + \bar{B})K] = \emptyset \ \forall K \in \mathbf{K}$ where $\mathbf{K}$ is the set of diagonal block matrices as indicated in equation (6).

As for remark (2), a sufficient criterion which can be handled more easily mathematically and numerically; an iterative algorithm is based on the gradient search technique that terminates when

(i) a limiting value $\varepsilon = |V^t - V^{-1}|$ approaches a small prespecified value.
(ii) matches left-right sides of equation (21) that zeroes the off diagonal blocks of equation (21) as indicated in equation (22).

Our procedure for (i) and (ii) is an "open loop" in the sense that if an unacceptable results due to, for instance, non orthogonal eigenvectors or nonzero off diagonal blocks for the decentralized feedback matrix K, other selections for the free parameters to improve the undesirable results is possible.

References

Huang, S. and S. Zhang, Comments on 'Design of decentralized control for symmetrically interconnected systems', submitted.

Kashlan, A. El and M. El Geneidy, (1996). Design of decentralized controls for symmetrically interconnected systems. *Automatica*, 32(3), 475–476.