INFLUENCE OF SURFACE COOLING ON EXTRUDATE SWELL -
A NUMERICAL STUDY

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ABSTRACT

Influence of convective cooling at the extrudate’s free surfaces on its own swelling behaviour is examined numerically when viscous fluids of a Newtonian type are extruded. In circular die extrusion, increasing the surface heat transfer coefficient $h$ increases extrudate’s thickness swell at low Nahe-Meritt number $Na$; at high $Na$, however, this trend tends to be reversed. With annular dies, a higher cooling rate also enhances thickness swell, but suppresses extrudate’s outward deflection. The influence of $h$ on extrudate swells is stronger in extrusion situation with higher Péclet number.

KEYWORDS: EXTRUSION, EXTRUDATE, SWELL, SURFACE COOLING, ANNULAR DIES, NUMERICAL STUDY, HEAT TRANSFER.

INTRODUCTION

This work is concerned with a numerical study of the influence of cooling of extrudate’s free surfaces on its own swelling behaviour in extrusion through circular and annular dies under steady conditions.

Extrusion is an important manufacturing process, and extruded products abound. In studying extrusion, the material to be extruded is often modelled as a viscous fluid. However, it is known that when such a fluid is extruded through a die, in general the extrudate does not have the same size as the die orifice. The amount of change in the extrudate size, known as swelling, is of significant theoretical and practical interest and has been investigated in many works [1-4, to name but a few]. Temperature has been seen to be among the many factors affecting extrudate swell. However, in only relatively few works that its influence as occurred after the die exit has been considered [5-8], while hollow extrudates resulting from extrusion through annular dies have attracted even less attention.

In an effort to provide further understanding of the extrudate swell phenomenon, a main aim of this work is thus to examine the effects of surface cooling on extrudate swell, particularly in annular extrusion.

MATHEMATICAL MODEL AND NUMERICAL METHOD

The die geometry considered is in the form of a straight, circular tube on the outside, and a concentric cylindrical mandrel with a conical head facing the flow on the inside, thus forming a uniform annular gap between them before the die exit. The arrangement is shown diagrammatically in Fig. 1; the chosen geometry should be representative enough of an extrusion process through annular dies; see, for example, Morton-Jones [9], and Benbow and Bridgwater [10] for some similar geometries.

The mathematical model used is that for a steady, axisymmetric, non-isothermal flow of incompressible, Newtonian fluids with free surfaces and without body forces. The governing equations are those of conservation of mass and momentum, and balance of energy [11]. A non-dimensionalisation scheme is also used and the non-dimensional parameters and variables are defined such that the form of the governing equations is unchanged [4].

The coupling between the flow and temperature fields is via the fluid’s temperature-dependent viscosity $\mu$, which is assumed to decrease exponentially with temperature $T$ according to the formula $\mu = \mu_0 e^{-\alpha T}$, where $\alpha$ is a non-dimensional exponential coefficient and $\mu_0$ a constant, here set equal to 1. Other fluid properties are assumed to be constant, and the following non-dimensional values are used: density $\rho = 1.67 \times 10^3$; thermal conductivity $k = 0.190$ ; specific heat capacity $c = 7.19 \times 10^3$. It was shown [4] that when appropriate values are taken for the physical parameters like temperatures, mean flow velocity, tube radius, etc., then the above non-dimensional properties correspond approximately to those of low-density polyethylene under zero shear at about 150°C - 190°C temperature range.

Boundary Conditions

Referring to Fig. 1 and using standard notations, the following non-dimensional boundary conditions are used:

(a) At entrance to the flow domain ($z = -5$): parabolic velocity profile and uniform, zero temperature;

$u = 0, \quad w = 2 \left(1 - r^2\right), \quad T = 0$

(b) Along the tube wall ($r = 1, \ z \leq 0$) and mandrel wall: non-slip condition and zero temperature;

$u = w = 0, \quad T = 0$

(c) Along the centre-line ($r = 0, \ z \leq 2$): zero radial velocity and shear stress, and axis-of-symmetry condition for temperature; $u = 0, \ r_{rz} = 0, \ \partial T / \partial r = 0$

(d) At the “far-downstream” section or exit of the flow domain (where $z$ is sufficiently large so that there is no further change to the extrudate dimensions; here values of $z = 4, 4.4$ and 6 have been used): zero radial velocity and axial stress; $u = 0, \ t_{zz} = 0$

No thermal boundary condition is imposed on this end, thus the solution attempts to make $\partial T / \partial z = 0$ here. This is acceptable, as discussed in Reference [1].

(e) On the free surfaces ($z > 0$): zero stresses;

$t_{nn} = t_{ns} = 0$

where $n$ is the (non-dimensional) outward-pointing coordinate normal to the surfaces, and $s$ is the co-ordinate along them. Convective cooling conditions are also imposed on the two free surfaces. Here, these are

$\frac{\partial T}{\partial n} = f_o (-0.72 T - 28.8)$ on the inner surface, and
RESULTS AND DISCUSSION

Two annulus ratios $R_{\text{outer, annul}}/R_{\text{inner, annul}}$ of 0.7 and 0.9, as well as the degenerative situation of capillary extrusion when the mandrel core is absent are considered under a combination of different $f_m$ and $f_{out}$ values. The combination results in a total of 13 “series” as listed in Table 1. The series are also grouped into 3 groups according to the annular gap size. Within each series, the fluid viscosity’s exponential coefficient $\alpha$ is varied, allowing the Nahme-Griffith number $Na$ to change as a primary changing parameter. The Nahme-Griffith number based on conditions at die exit is defined as

$$Na = \alpha W \mu / k$$

where $W$ is the mean velocity in the axial direction. The Pécel number at die exit, defined as

$$Pe = \rho c W (R_o - R_i) / k$$

is, however, constant within each series; here $R$ stands for radius, and subscripts $o$ and $i$ indicate outer and inner radii respectively.

The local Reynolds number based on a local length scale $L$ and defined as $Re = \rho W L / \mu$ is always very small. Using the annular gap size for $L$ and the minimum value attained near the die exit for $\mu$, the maximum $Re$ attained is about $1 \times 10^{-6}$.

Extrudate Swells

To characterise swelling in extrusion through annular dies, the following two swell ratios are used (subscripts $extrudate$ and $exit$ indicate the conditions at the “far-downstream” section of the extrudate and at the die exit, respectively):

- Extrudate thickness swell ratio
  $$s_t = \frac{[\text{extrudate thickness} - \text{exit gap size}]}{\text{exit gap size}}$$
  $$= \frac{[R_o - R_i]_{\text{extrudate}} - (R_o - R_i)_{\text{exit}}}{(R_o - R_i)_{\text{exit}}}$$
  Mean radius swell ratio
  $$s_m = \frac{[R_m]_{\text{extrudate}} - (R_m)_{\text{exit}}}{(R_m)_{\text{exit}}}$$

where $R_m$ is the mean radius given by $R_m = (R_o + R_i) / 2$. Thus $s_m$ provides a measure of the extrudate’s radial deflection relative to the die exit location.

Fig. 2 shows variation of $s_t$ in terms of the changing heat transfer coefficient $h$ and $Na$ for all 13 series of the 3 groups. For the series of groups $B$ and $D$, when both surfaces have the same heat transfer coefficient $h$, increasing $h$ results in larger $s_t$. Since a higher $h$ causes cooler, hence more viscous, surface layers of the extrudate relative to its middle region, this swelling behaviour is thus consistent with a theory due to Tanner [13]. At low $Na$, group $O$ also displays an increase in $s_t$ as $h$ is increased, consistent with what has been observed in some previous works [6-8]. However, at higher $Na$, this pattern is upset and tends to be reversed.

When the extrudate’s inner and outer free surfaces are subjected to significantly different cooling rates, it seems that some averaging effects take place. Thus results of series $B20$ are virtually the same as $B11$, while those of series $D20$ and $D02$ are nearly identical with $D11$. Results from series $B02$ also show these averaging effects and are very similar to $B11$, but only at low $Na$. At higher $Na$, however, series $B02$ shows a strong increase in $s_t$ such that by the end of the $Na$ range considered, a value of $s_t$ of 110.8 % is reached. This is perhaps due to a significantly larger outer surface area which, when strongly cooled, enhances swelling as per Tanner’s theory. And this swelling dominates over any effect on $s_t$ of the adiabatic, smaller inner surface; the net result is a large $s_t$ value.

Fig. 2 also shows that the effects of a changing heat transfer coefficient are most pronounced in group $O$ which has largest $Pe$, and least, to near insignificance, in group $D$ which has smallest $Pe$. While it is rather surprising that $h$ has such insignificant influence in group $D$, the pattern is not unexpected. Since, for example, when $Pe$ is large, conductive effects will be relatively small and as a result relatively large temperature differences exist between the extrudate’s layers. This would lead to a stronger influence of $h$ on $s_t$ as seen, for example in group $O$.

Fig. 3 shows variation of $s_m$ in terms of $h$ and $Na$ for the series of groups $B$ and $D$. Again, changes in group $B$ are more pronounced, and this is consistent with $Pe$ being higher in this group. When both surfaces have similar $h$, a higher $h$ reduces $s_m$ which is a measure of the extrudate’s outward deflection relative to die exit’s location. That this deflection should decrease with a higher cooling rate agrees with expectation, since a higher cooling rate gives rise to a cooler and more viscous extrudate, which would resist any radial displacement more effectively. When different $h$ values are used on the two free surfaces, larger difference in surface temperatures results and causes extra deflection of the extrudate toward the cooler side. Thus for group $D$, $s_m$ is largest with series $D02$, and least with series $D20$. This pattern is also seen in group $B$, but only at low $Na$. At higher $Na$, the pattern is reversed. At the highest $Na$ value used here, series $B02$ has produced the lowest $s_m$, namely an inward swing of about 3.5 %.

Finally, Fig. 4 shows an example case of extrudate shape showing large thickness swell, and the corresponding temperature distribution.

CONCLUSIONS

Influence of convective cooling at the extrudate’s free surfaces on its own swelling behaviour has been examined numerically when visous fluids of a Newtonian type are extruded through annular and circular dies.

For circular dies, higher surface cooling rate enhances thickness swell in agreement with some previous works, but only when $Na$ is small. At larger $Na$, this trend tends to be reversed. With annular dies, higher cooling rate always enhances thickness swell, consistent with a previous theory, but tends to suppress extrudate’s radial deflection.
Higher Péclet number is also seen to enhance the effects of surface cooling on extrudate swells.

REFERENCES

<table>
<thead>
<tr>
<th>Group</th>
<th>Annular gap radii</th>
<th>Case series</th>
<th>( f_{in} / f_{out} )</th>
<th>Grid pattern Used (grid points)</th>
<th>Péclet number Pe</th>
</tr>
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<tbody>
<tr>
<td><strong>B</strong></td>
<td>0.7 - 1</td>
<td>B00</td>
<td>0 / 0</td>
<td>( 14 \times 99 )</td>
<td>37.1</td>
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<tr>
<td></td>
<td>(Gap size 0.3)</td>
<td>B11</td>
<td>1 / 1</td>
<td>( 14 \times 99 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B22</td>
<td>2 / 2</td>
<td>( 14 \times 99 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B02</td>
<td>0 / 2</td>
<td>( 14 \times 99 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B20</td>
<td>2 / 0</td>
<td>( 14 \times 99 )</td>
<td></td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>0.9 - 1 (Gap size 0.1)</td>
<td>D00</td>
<td>0 / 0</td>
<td>( 14 \times 99 )</td>
<td>33.2</td>
</tr>
<tr>
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<td>1 / 1</td>
<td>( 14 \times 99 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>D22</td>
<td>2 / 2</td>
<td>( 14 \times 99 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>D02</td>
<td>0 / 2</td>
<td>( 14 \times 99 )</td>
<td></td>
</tr>
<tr>
<td></td>
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<td>D20</td>
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<td><strong>O</strong></td>
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<td>O0</td>
<td>– / 0</td>
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<td></td>
<td>O1</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>O2</td>
<td>– / 2</td>
<td>( 12 \times 82 )</td>
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Table 1. Cases considered.

Figure 1. Model of the flow domain and die geometry.

Figure 2. Variation of \( s_{st} \) with respect to \( Na \) for all series.

Figure 3. Variation of \( s_{sm} \) with respect to \( Na \) for all series.

Figure 4. An example pattern of temperature distribution, from the case with \( Na = 14.14 \) of series \( B02 \).