
9.149. The flowrate is 0.13 m³/s without the pump. Calculate the approximate pump power required to maintain a flowrate of 0.17 m³/s.

We first apply work-energy equation to determine head loss in flow without pump:

\[ v_{150} = \frac{Q}{A} = \frac{0.13}{\frac{\pi}{4} (0.15)^2} = 7.36 \text{ m/s} \]

\[ v_{75} = \frac{Q}{A} = \frac{0.13}{\frac{\pi}{4} (0.075)^2} = 29.43 \text{ m/s} \]

\[ z_1 + \frac{v_1^2}{2g} + \frac{P_1}{\gamma} = z_2 + \frac{v_2^2}{2g} + \frac{P_2}{\gamma} + h_L \]

\[ 60 + 0 + 0 = 0 + \frac{v_{75}^2}{2g} + 0 + f \frac{L}{D} \frac{v_{75}^2}{2g} \]

\[ f \frac{L}{D} = 5.743 \]

We assume \( f \frac{L}{D} \) is the same for the flow with pump

\[ Q = 0.17 \text{ m³/s} \]

\[ v_{150} = \frac{Q}{A} = \frac{0.17}{\frac{\pi}{4} (0.15)^2} = 9.62 \text{ m/s} \]

\[ v_{75} = \frac{Q}{A} = \frac{0.17}{\frac{\pi}{4} (0.075)^2} = 38.48 \text{ m/s} \]

Apply work-energy again:

\[ E_p + 60 = \frac{v_{150}^2}{2g} + f \frac{L}{D} \frac{v_{150}^2}{2g} = 75.5 + (5.743)(4.72) \]

\[ E_p = 42.6 \text{ m} \]

\[ \text{Power} = Q \gamma E_p = (0.17)(9.807)(42.6) = 70.9 \text{ kW} \]
A horizontal 50 mm PVC pipeline leaves a square-edged entrance a water tank 3 m below its free surface. At 15 m from the tank, it enlarges abruptly to a 100 mm pipe which runs 30 m horizontally to another tank, entering it 0.6 m below its surface. Calculate the flow rate through the line (water temperature 20°C), including all head losses.

From App. 2, water at 20°C

\[ V = 1.003 \times 10^{-6} \, m^2/s \]

Pipe 1 has diameter 50 mm
Pipe 2 has diameter 100 mm

By continuity,

\[ V_1 = \left(\frac{100}{50}\right)^2 V_2 = 4 V_2 \]

Reference: Text

7th ed., 1996

Compute \(R\) in terms of velocity

\[ R_1 = \frac{V_1 d_1}{V} = \frac{V_1 (0.05)}{1.003 \times 10^{-6}} = 99850 \, V_1 = 199400 \, V_2 \]

\[ R_2 = \frac{V_2 d_2}{V} = \frac{V_2 (0.10)}{1.003 \times 10^{-6}} = 99700 \, V_2 \]

The local losses we must include are:

Square edged entrance: \(h_{xc} = 0.5 \frac{V_1^2}{2g_n}\) (Fig. 9.17)

Sudden expansion: \(h_{lex} = \frac{(V_1 - V_2)^2}{2g_n}\) (9.17)

Exit to Reservoir: \(h_{lx} = \frac{V_2^2}{2g_n}\)

Frictional losses we write simply as

\[ h_{fe1} = f_1 \frac{V_1^2}{d} \frac{V_1^2}{2g_n} = f_1 \left(\frac{15}{0.05}\right) \frac{V_1^2}{2g_n} \]

\[ h_{fe2} = f_2 \frac{V_2^2}{d} \frac{V_2^2}{2g_n} = f_2 \left(\frac{30}{0.1}\right) \frac{V_2^2}{2g_n} \]

at both locations

We can now apply Eqn 7.35 between the surfaces of the two reservoirs:

\[ z_1 = z_2 + h_{xc} + h_{lex} + h_{lx} + h_{fe1} + h_{fe2} \]
\[ 3 = 0.6 + 0.5 \frac{V_1^2}{2g_n} + \frac{(V_1 - V_2)^2}{2g_n} + \frac{V_2^2}{2g_n} + f \left( \frac{15}{0.05} \right) \frac{V_1^2}{2g_n} + f_2 \left( \frac{30}{0.1} \right) \frac{V_2^2}{2g_n} \]

We can replace \( V_1 \) with \( 4V_2 \) and rearrange to the following form:

\[ V_2 = \frac{6.86}{\sqrt{18 + 4800 f_1 + 300 f_2}} \] (a)

At this point, a trial and error solution is required. First assume \( V_2 \), calculate \( R_1 \) and \( R_2 \), obtain \( f_1 \) and \( f_2 \) from Fig 9.10 (assuming PVC is smooth), then recompute \( V_2 \) using Eqn (a). Repeat until the assumed \( V_2 \) matches the computed \( V_2 \).

\[ V_2 = 0.67 \text{ m/s} \]
\[ R_2 = 66.691 \rightarrow f_2 = 0.0145 \]
\[ R_1 = 133381 \rightarrow f_1 = 0.017 \]

\[ Q = 0.0053 \text{ m}^3/\text{s} = 5.3 \text{ l/s} \]