1. In the pumping system shown, water is pumped from a reservoir and discharged into the atmosphere. The inlet pipe from the reservoir to the pump has length $L_1 = 1000$ m and diameter $d_1 = 0.220$ m. On the discharge side, the pipe has length $L_2 = 500$ m and diameter $d_2 = 0.180$ m. Both pipes are of clean cast iron. For $H = 5$ m, find the water flow rate and the pump power when cavitation is about to begin at the pump inlet. Water temperature is $20^\circ$ and barometric pressure is 100 kPa. Make reasonable assumptions.

2. Determine the total water flow rate that is being drained from a large tank, as shown. All pipes are of commercial steel type. The valves are fully open and both have a loss coefficient of $K = 10$. Water temperature is $20^\circ$ C. Make reasonable assumptions. Other data:
   - Pipe 1: length 200 m, diameter 100 mm
   - Pipes 2 & 3 (Identical): length 100 m, diameter 60 mm

   (Ans: $0.0377 \text{ m}^3/\text{s}$, $6.24 \text{ kW}$)

   (Ans: $0.0156 \text{ m}^3/\text{s}$
   
   $= 15.6 \text{ l/s}$)
Kerovitic pump: \[ P_0 = P_t = 2340 \text{ Pa} \]

1. Cast iron inlet pipe
   - 1000 m, 0.22 m dia.
2. Cast iron discharge pipe
   - 500 m, 0.18 m diameter

\[ \nu = 20^\circ C \]

Cast iron: \( e = 0.26 \text{ mm} \)

(wall roughness height)

\[ P_t = \nu = 2340 \text{ Pa} \]

\[ \frac{100000}{1000 \times 9.8} + \frac{0}{2g} + 0 = \frac{2340 + \frac{V_1^2}{2g}}{1000 \times 9.8} + \frac{5 + f(100000)}{10 \times 9.8} \]

\[ \frac{V_1^2}{2g} \left(1 + 4.555 \frac{2}{11} \right) = 4.965 \]

\[ V_1^2 \left(1 + 4.555 \frac{2}{11} \right) = 97.3 \]

\[ L = 1000 \Rightarrow V_2^2 \left(1 + 4.555 \frac{2}{11} \right) = 97.3 \]

\[ e = 0.26 \text{ mm}, \quad \text{relative roughness } \frac{e}{D} = \frac{0.26}{220} = 0.0012 \]

Assuming \( f = 0.020 \Rightarrow V_2 = 1.029 \text{ m/s} \)

\[ \text{Re} = \frac{V_2 D}{\nu} = \frac{1.029 \times 0.18 \times 10^{-6}}{1.05 \times 10^{-6}} = 2.3 \times 10^5 \]

 Moody diagram \( \Rightarrow \) (with \( \text{Re} = 2.3 \times 10^5, \frac{e}{D} = 0.0012 \))

\[ f = 0.0215 \]

Now try \( f = 0.0215 \Rightarrow V_2 = 0.993 \text{ m/s}, \text{Re} = 2.2 \times 10^5 \Rightarrow f = 0.0215 \]

O.K.
Hence, \( V_E = 0.993 \) m/s

Flow rate \( Q = \frac{\pi}{4} \times 0.12^2 \times 0.993 \approx 0.0377 \) m³/s \( (\checkmark) \)

\( (37.7 \text{ l/s}) \)

\[ \frac{Q}{
u} + \frac{Q}{
u} + 5 = \frac{V_E^2}{2g} + 5 + f_1 \left( \frac{L_1}{D_1} \right) \frac{V^2}{2g} + f_2 \left( \frac{L_2}{D_2} \right) \frac{V^2}{2g} \]

\[ Q = \frac{V_E}{4} \quad V_1 = \frac{V_E}{D_2} \quad V_E \Rightarrow V_E = \frac{V_1 D_2}{D_2^2} = \frac{0.993}{0.18} \]

\[ h_e = \frac{V_E D_2}{\nu} = \frac{1.483 \times 0.18}{1 \times 10^{-6}} = 2.7 \times 10^5 \]

\[ \text{in pipe 2:} \quad \varepsilon_2 = \frac{0.26}{190} = 0.0014 \]

\[ f_2 = 0.022 \]

\[ h_{\text{pump}} = \frac{1.483}{2 \times 9.8} \left[ 1 + 0.022 \times \frac{500}{0.18} \right] + 5 + 0.0215 \frac{1000 \times 0.993^2}{0.22} + 1.98 \]

\[ = 16.9 \text{ m} \]

Power \( P = g \cdot Q \cdot h_{\text{pump}} \approx 9.8 \times 1000 \times 0.0377 \times 16.9 \]

\[ = 6240 \text{ W} = 6.24 \text{ kW} \ (\checkmark) \]
\[ Q_1 = Q_2 + Q_3 = 2Q_2 \]

\[ V_1d_1^2 = 2V_2d_2^2 \Rightarrow V_1 = 2V_2 \left( \frac{d_1}{d_2} \right)^2 = 2V_2 \left( \frac{0.06}{0.10} \right)^2 \]

Bernoulli: \( A \rightarrow C \) (identical to \( A \rightarrow B \))

\[ 25 = \frac{V_2}{2g} + f_1 \left( \frac{100}{0.1} \right) \left( \frac{0.72V_2}{2g} \right)^2 + 0.5 \left( \frac{0.72V_2}{2g} \right) + \left( \frac{0.72V_2}{2g} \right)^2 \]

\[ \text{valve loss} \]

\[ V_2 = \left[ \frac{25 \times 2g}{1 + 0.5 \times 0.72^2 + 0.72^2 + 10 + f_1 \frac{100}{0.1} \frac{200 \times 0.72^2}{2g} + f_2 \frac{100}{0.06}} \right]^{1/2} \]

Commercial Steel pipes: \( \varepsilon = 0.046 \text{ mm} \)

\[ \varepsilon_1 = \frac{0.046}{100} = 0.00046 \]

\[ \varepsilon_2 = \frac{0.046}{60} = 0.00077 \]

Diameter: \( f_1 = 0.0165 \); \( f_2 = 0.018 \) \( \Rightarrow V_2 = 2.88 \text{ m/s} \)

\[ \text{Re}_1 = \frac{1000 \times 0.72 \times 2.88 \times 0.1}{0.001} = 2.0 \times 10^5 \Rightarrow f_1 = 0.0187 \]

\[ \text{Re}_2 = \frac{1000 \times 2.88 \times 0.06}{0.001} = 2.5 \times 10^5 \Rightarrow f_2 = 0.0195 \]

\[ V_2 = 2.77 \text{ m/s} \Rightarrow \text{Re}_1 = 2.0 \times 10^5; \text{Re}_2 = 2.4 \times 10^5 \]

\( f_1 = 0.0190 \); \( f_2 = 0.020 \)

\[ V_2 = 2.75 \text{ m/s} \Rightarrow \text{Re}_1 = 1.98 \times 10^5; \text{Re}_2 = 2.3 \times 10^5 \]

\( f_1 = 0.0191 \); \( f_2 = 0.020 \) (rounded)

\[ V_2 = 2.75 \text{ m/s} \Rightarrow V_1 = 0.72V_2 = 2.0154 \text{ m/s} \]

Total flow rate \( Q = \frac{4}{3} \pi d_1^2 V_1 = \frac{\pi}{4} \times 0.1^2 \times 1.98 = 0.0154 \text{ m}^3/\text{s} \]

\[ = 15.6 \text{ l/s} \]