\[
\frac{\partial b}{\partial t} + \nabla \cdot (v b) = \frac{\partial b}{\partial t} + \frac{\partial}{\partial x} \left( \frac{\partial b}{\partial x} \right) = 0
\]
If \( p = \text{constant} \), then

\[
\rho = \frac{8g}{c^2} + C \text{ is constant.}
\]

Thus, the sum of the constant pressures,

\[
\sum \frac{8g}{c^2} = \text{constant.}
\]

For \( p = \text{constant} \),

\[
\rho = \frac{8g}{c^2} + C
\]

integrating, we obtain

\[
\sum \frac{8g}{c^2} + p = \text{constant.}
\]

From (3), can be separated to give

\[
\sum \frac{8g}{c^2} + p = \text{constant},
\]

and \( c = 0 \)

\[
\sum \frac{8g}{c^2} = \text{constant.}
\]

uniform circular motion about a vertical axis.
Example (C. Flore)

[Diagram of a U-tube with dimensions and pressures]

2.122) Given:
U-tube as shown:
\( a_x = 3 \text{ m} \)
\( a_y = 3 \text{ m} \)
No spillage of liquid.

To find: \( P_a, P_b \)
for:
1) Water
2) Mercury

Solution:
For \( P = \rho g h \), where \( h = \) vertical distance from free surface.

a) for water:
\[
P_a = \rho g h = \rho g y_a = (9.789 \text{ KN/m}^3)(1.5) = 14.68 \text{ KN/m}^2
\]
\[
P_a = \rho g h_a = \rho g y_a = \rho g (1.5 - 0.3 - 0.3 \tan 45^\circ)
\]
\[
4 = \tan^{-1}\left(\frac{a_x}{a_y + g}ight) = \frac{3}{0.472} = 17^\circ
\]
\[
P_a = 9.789 (1.5 - 0.3 - 0.3 \tan 17^\circ) = 10.85 \text{ KN/m}^2
\]
\[
P_b = \rho g h_b = \rho g y_b
\]
\[
= 9.789 (1.5 - 0.3 + 0.3 \sin 45^\circ - 0.3 \cos 45^\circ \tan 45^\circ)
\]
\[
P_b = 13.19 \text{ KN/m}^2
\]

b) for \( Hg \):
Since \( P_a \) r.d. for \( Hg \) r.d. = 13.57
\[
P_a = 10.85 (13.57) = 147.05 \text{ KN/m}^2
\]
\[
P_a = 14.68 (13.57) = 199.21 \text{ KN/m}^2
\]
\[
P_b = 13.19 (13.57) = 178.99 \text{ KN/m}^2
\]
Example

A tank containing water accelerates down a slope as shown. Find the slope of water's free surface with respect to the horizontal.

(from Gerharz et al.)

Tank: rectangular cross section, 10 cm long, 6 m height.
Half filled with water originally; accelerates down the slope with \( a = 1 \text{ m/s}^2 \).

Solution

Resolving the acceleration \( a \) into the \( x \) & \( z \) components as shown, then

\[
\begin{align*}
    a_x &= a \cos 20^\circ = 1 \times \cos 20^\circ = 0.9397 \text{ m/s}^2 \\
    a_z &= -a \sin 20^\circ = -1 \times \sin 20^\circ = -0.342 \text{ m/s}^2
\end{align*}
\]

The slope of the water free surface is given by

\[
\frac{dz}{dx} = -\frac{a_x}{a_z + g} = -\frac{0.9397}{0.342 + 9.8} = -0.09936
\]

\[
\theta = \tan^{-1} 0.09936 = 5.67^\circ.
\]

\[
\theta = 5.67^\circ
\]
Alternatively, it can be seen that the new free surface (or any surface of constant pressure) is perpendicular to the vector \( \vec{a} \) which is the sum of vector \( \vec{g} \) and vector \( -\vec{a} \), as shown below.
An example.

A closed cube, each side 300 mm long, has a small opening at the centre O of its top surface (which is horizontal). When it is filled with water, and rotated uniformly about a vertical axis through O at 1000 rpm, find the force on a side face due to water.

.../2

Q.4 An example.

If the mercury U-tube shown is rotated about a vertical axis through O, calculate its rotational speed when the difference of level between its two legs is 20 cm. Determine the pressure difference $P_A - P_B$. What is the original height of the mercury column? Is the free surface as shown.

END.
Referring to the attached question paper,

Now, the free surface is given by

\[ z = \frac{c}{2g} \left( \frac{w}{2} + c \right) \]

\[ \text{on leg A: } z_a = \frac{c}{2g} \left( \frac{w}{2} + c \right) \]
\[ \text{on leg B: } z_b = \frac{c}{2g} \left( \frac{w}{2} + c \right) \]

And as \( z_a - z_b = 0.2 \) m is given,

\[ 0.2 = \frac{w}{2g} \left( 0.4 - 0.2 \right) \]
\[ \Rightarrow w = \frac{0.2 \times 2g}{0.4 - 0.2} \]
\[ = \frac{0.2 \times 2g}{0.2} \]
\[ = 3.267 \text{ m/s} \]

\[ w = \frac{5.72}{1.74} \text{ rad/s} \]

The pressure at any location \((x, z)\) is given by

\[ p = -\gamma z + \frac{\rho u^2}{2} + p_0 \]

where \( p_0 = \text{atm} \), \( z = 0 \)

Take the origin at 0, \( z \) is the vertical distance from the Origin, the upward.

Now the coordinates of A and B are

A: \( z = 0 \), \( r = 0 \) m
B: \( z = 0 \), \( r = 0.2 \) m
Now, axis of rotation is at through A (here coordinate of A: x_A, y_A, z_A)

Now \( \frac{p}{A} + \frac{T}{2g} = 0 \)

Hence, the smallest pre-cure occurs when \( r = 0 \) (\( r \geq 0 \) always) and longest \( z \) above A

- location of first contact is at \( D/Q \) as shown

- At initiation of contact:
  \[ p_D = \frac{\text{Brane}}{2}\cos \theta \]
  \[ = \frac{2.34 m}{2} \cdot 2 \cos 20^\circ \]
  \[ = 2.34 \cdot 2 \cos 20^\circ \]
  \[ = 2.34 \cdot 1.93 \]
  \[ = 4.62 \text{ m} \]

- Letting \( p = p_A \):
  \[ p_A = 2.34 \cdot 2 \cos 20^\circ \]
  \[ = 4.62 \text{ m} \]

Coordinates of corridor:
  - \( x_c = 0.8 \text{ m} \)
  - \( y_c = 0.48 \text{ m} \)

- \[ 101300 = 6260 + \frac{1000 \times 4.7 \times 2 \times 0.8^2}{2 \times 9.8} - 1000 \times 0.03 \]
  \[ = 17.6 \times 60 = 168 \text{ rev/min} \]

\( \omega = 309.25 \text{ rad} \)

Thus

\[ \begin{align*}
P_A &= \sigma + \rho g \omega^2 r_A^2 + P_0 \\
\rho g &= 2 \left( P_B - P_A \right) \\
P_B &= \sigma + \rho g \omega^2 r_B^2 + P_0
\end{align*} \]

Alternatively, on the A leg:

\[ \begin{align*}
P_A &= \frac{1}{2} \rho g \omega^2 \left( r_A^2 - r_B^2 \right) + \frac{\rho \omega^2 \omega r_A^2}{2} \\
P_B &= \frac{1}{2} \rho g \omega^2 \left( r_B^2 - r_A^2 \right) + \frac{\rho \omega^2 \omega r_B^2}{2}
\end{align*} \]

\[ \begin{align*}
P_B &= \frac{1}{2} \rho g \omega^2 \left( r_B^2 - r_A^2 \right) + \frac{\rho \omega^2 \omega r_B^2}{2}
\end{align*} \]

\[ \begin{align*}
P_A &= \frac{1}{2} \rho g \omega^2 \left( r_A^2 - r_B^2 \right) + \frac{\rho \omega^2 \omega r_A^2}{2}
\end{align*} \]

\[ \begin{align*}
P_A &= \frac{1}{2} \rho g \omega^2 \left( r_A^2 - r_B^2 \right) + \frac{\rho \omega^2 \omega r_A^2}{2}
\end{align*} \]

\[ = 2.67 \times \frac{10^3}{\text{N/m}^2} \]

\[ = 26.7 \text{ kPa} \]

What is the original height of the mercury column?

(Given that the parabola connecting the two free surfaces also passes through \( O \))

\[ \text{Free on leg } A - \text{Free on leg } B = 0 \]

\[ \begin{align*}
P_A &= \rho g \omega^2 \left( r_A^2 - r_B^2 \right) + P_0 \\
P_B &= \rho g \omega^2 \left( r_B^2 - r_A^2 \right) + P_0
\end{align*} \]

Write this equation for the two free surfaces,

\[ \begin{align*}
P_A &= \rho g \omega^2 \left( r_A^2 - r_B^2 \right) + P_0 \\
P_B &= \rho g \omega^2 \left( r_B^2 - r_A^2 \right) + P_0
\end{align*} \]

\[ \begin{align*}
\frac{1}{2} (l_A + l_B) &= \frac{\rho g \omega^2 (r_A^2 - r_B^2)}{2} \\
\text{Original height} &= \frac{1}{2} (l_A + l_B)
\end{align*} \]

\[ \begin{align*}
l_0 &= 0.10 \text{ m}
\end{align*} \]
(a) The cylinder in Fig. P2.137 accelerates to the left at a rate of 9.80 m/s². Find the tension in the string connecting the rod of circular cross section to the cylinder. The volume between the rod and the cylinder is completely filled with water at 10°C.

![Diagram of cylinder and string](image)

Figure P2.137

Given: g = 9.80 m/s²

(a) No tension

(b) Find the tension in the string of part (a) if the rod has a specific gravity S = 0.75.

Given, Fig. P2.137. Cylinder accelerates to left at rate of 9.80 m/s². Fluid is water at 10°C.

Find: Tension in string.

Solution. First find the pressure difference in the water over a length L = 8.0 cm. Since gravity is perpendicular to the rod, Eq. (2.4) gives

\[ dP = -\rho g dx \]

for the x-direction. Integrating gives

\[ \rho_x - \rho_i = -\rho g (x_2 - x_1) \]

For 10°C water, Table A.5 gives

\[ \rho_x - \rho_i = (1000 \frac{kg}{m^3})(9.80 \frac{m}{s^2})(8.0 \text{ cm})(100 \text{ cm}) = 782 \text{ N/m}^2. \]

We next apply Newton's second law to the rod

\[ \sum F_x = ma_x \]

or

\[ T + (\rho_x - \rho_i)A = ma_x \]

or

\[ T = (\rho_x - \rho_i)A \]

Assuming the string is not elastic, \( a_x \text{rod} = 9.80 \text{ m/s}^2 \).


\[ \text{Now} \]

\[ m = \rho \text{rod} \cdot A \cdot L = 0.0126 \text{ kg} \]

and

\[ A = \frac{\pi d^2}{4} = \frac{\pi}{4} (1.0 \text{ cm})^2 \left( \frac{m}{100 \text{ cm}} \right)^2 = 7.85 \times 10^{-5} \text{ m}^2 \]

Then
PROBLEM 2.137

\[ T = (-784 \text{ N/m}^2)(7.854 \times 10^{-5} \text{ m}^3) + (0.0126 \text{ kg})(9.8 \text{ m/s}^2) \]

\[ T = 0.062 \text{ N.} \]

(mq)

---

PROBLEM 2.138

Given: Fig. P2.137. Cylinder accelerates to left at rate of 9.80 m/s². Fluid is water at 10°C. \( S_{ad} = 0.75 \).

Find: Tension in string.

Solution: This solution is identical to Problem 2.137 where the equation for tension was found to be

\[ T = (p_2 - p_1) + MQ \]

where

\[ p_2 - p_1 = -784 \text{ N/m}^2, \quad A = 7.854 \times 10^{-5} \text{ m}^2, \quad \rho_k = 9.80 \text{ m/s}^2; \]

and

\[ M = \frac{\pi}{\rho_k} S_{ad} L \rho \approx \frac{(1000 \text{ kg/m}^3)(0.75)(10 \text{ cm})}{2}(0.01 \text{ cm})^3(\frac{1}{100 \text{ cm}})^3 \]

\[ = 0.00471 \text{ kg.} \]

Then

\[ T = (-784 \text{ N/m}^2)(7.854 \times 10^{-5} \text{ m}^3) + (0.00471 \text{ kg})(9.8 \text{ m/s}^2) = 0.0154 \text{ N.} \]

There is no tension in the string.

(mq)
The U-tube shown contains water at 20°C, and rotates around the axis \( z-z \). Determine the rotational speed in rpm (revolutions per minute) that will cause the water to start to vaporise at the bottom of the tube (point A).

Let the origin of axes be at A, then the free surface B on a tube leg has coordinates

\[ r = 0.10\, \text{m}, \quad z = 0.8\, \text{m} \]

When the water at A starts to vaporise,

\[ p_A = p_{\text{vapour}} = 2.34 \times 10^5 \, \text{Pa} \]

Now \( p_B = \text{ambient, always} \)

\[ p_B = 101300 \, \text{Pa at standard conditions} \]

Now \( p_B \) at \((r, z)\) = \[ \frac{1}{2} \rho g r^2 \, dz + p_B \]

where \( p_B \) = pressure at origin

\[ p_B = \frac{1}{2} \rho g r^2 \, dz + p_A \]

or

\[ 101300 = \frac{1}{2} (1000) g (0.1)^2 (1000 \times 9.8) (0.8) + 2340 \]

\[ \therefore \, \omega = 146.15 \, \text{rad/s} \]

or

\[ \omega = 146.15 \times 60 = 8790 \, \text{rpm} \]
1. The rectangular container shown is moving up the slope and gaining speed at a rate of 4 m/s². It is completely filled with water, and has a small opening at one of the top corners, as shown.

(a) Determine the location in the container where the maximum pressure occurs, and the value of this pressure.

(b) Calculate the total force on the back wall AB of the container due to the water.

Data: α = 20°, l = 3 m, h = 1 m, width of container w = 1.2 m (dimension into the page).

(Assumed load: 31.9 kN)

![Diagram of container with small opening and force diagram]

2. A vertical cylindrical tank 1.5 m high and 0.9 m in diameter is filled with water to a depth of 1.2 m. The tank is then closed and the pressure in the space above the water raised to 69 kPa. Calculate the pressure at the intersection of wall and tank bottom when the tank is rotated about a central vertical axis at 150 r/min. Also calculate the resultant force exerted by the water on the bottom of the tank.

(Assumed: P = 96.6 kPa, F = 53.4 kN)