Fluid Mechanics - Pressure - Momentum

**Problem:**

m. K. C. 104 F. C. 104

\[
\left( \frac{2}{n} \right)^2 - \left( \frac{2}{n} \right)^2 = n
\]

\[
m = D
\]

\[
g = \sqrt{n - \frac{2}{n}}
\]

\[
\text{For all standard conditions}
\]

**Diagram:**

[Diagram showing fluid flow and pressure distribution]

**Notes:**

- Reflect the fluid's mass and pressure differentials for the scenario, which is at 1000 K. Assume the fluid is operated at a temperature of 1200 and 1800 K. To determine flow at different velocities, set up the conditions for the respective cases. Use the momentum equation to analyze the fluid motion.

- The sensor is placed at an angle, and the fluid's flow is shown. The fluid's motion is depicted by arrows indicating the flow direction.

- The diagram illustrates the fluid's pressure distribution along the pipeline.
Take the control volume as shown.

Then applying momentum principle to it gives:

$$\sum F_x = \sum \rho V_x (V_x A)$$

and

$$\sum F_y = \sum \rho V_y (V_y A)$$

(with $F$ being assumed direction of force by bend or fluid)

or

$$-p_1 A_1 + F_x = \rho I A_2 \cos a$$

$x$-direction:

$$\rho (-V_i) (-V_i A_1) + \rho (V_2 \cos a) (V_2 A_2)$$

and

$y$-direction (with $W$ being weight of fluid in the control volume)

$$F_y = W - \rho I A_2 \sin a = \rho (\cos V_1 A_1) + \rho (V_2 \sin a) (V_2 A_2)$$
Bernoulli Eqn. between $\text{(1)}$ and $\text{(2)}$, assuming ideal flow, yields:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$Q = 0.070 \text{ m}^3/\text{s} \Rightarrow V_1 = \frac{0.070}{\frac{\pi}{4} \times 0.16} = 3.48 \text{ m/s}$

$$V_2 = \frac{0.070}{\frac{\pi}{4} \times 0.060^2} = 24.76 \text{ m/s}$$

$z_2 - z_1 = 1 \text{ m}$

$$\frac{P_1}{\gamma} = 1 + \frac{24.76^2}{2 \times 9.8} - \frac{3.48^2}{2 \times 9.8} = 31.66 \text{ m}.$$ $P_1 = 31.66 \times 1000 \times 9.8 = 310,300 \text{ Pa} = 310 \text{ kPa}$.

Hence,

$$A_1 = \frac{\pi}{4} \times 0.16^2 = 0.0201 \text{ m}^2; A_2 = \frac{\pi}{4} \times 0.06^2 = 0.002827 \text{ m}^2$$

$$= 310,300 \times \left(\frac{\pi}{4} \times 0.16^2\right) + F_x = 1000 \times (3.48)^2 (0.0201) + 1000 \times (24.76 \times 0.0660) + (24.76 \times 0.002827)$$

$$F_x = 7350 \text{ N} \rightarrow$$

Similarly:

$$F_y = 0.020 \times 1000 \times 9.8 = 1000 \times (24.76 \times 0.0660) \times \sin 60^\circ \times (24.76 \times 0.002827)$$

$$F_y = 4850 \text{ N} \downarrow$$
Net force on bolts:

\[
\begin{align*}
(F_x) &= 7350 \text{ N} \\
(F_y) &= 1873 \downarrow + 60 \times 9.8 \downarrow \text{ N} \\
\Rightarrow (F_y) &= 2462 \downarrow \\
\end{align*}
\]

Total force:

\[
F = \frac{7350}{14.4^2} \\
\theta = 18.5^\circ \\
\Rightarrow (F) = 7750 \text{ N}
\]
Problem 6.65

Taking the control volume as shown (bounded by faces 1, 2, and nozzle walls).
Momentum principles give:

- In the y direction: no forces.

\[ \sum F_y = \sum (V_y A_y) \]

\[ p_1 A_1 - p_2 A_2 + F_{\text{nozzle}} = p_1 V_1 (V_y A_y) + p_2 V_2 (V_y A_2) \]

But \( V_A = \dot{m} \) and \( \dot{m} V_A = \dot{m} \) (mass flow rate constant)

\[ p_1 A_1 - p_2 A_2 + F_{\text{nozzle}} = \dot{m} (V_y A_y) \]

\[ p_1 = p_2 = 0.5 V_y = 270 \text{ m/s} \]

\[ V_y = \text{sone speed} = \sqrt{\frac{k R T_2}{\text{gauge}}} \quad \text{(Vennard & Street, p. 13)} \]

\[ p_1 = 14 \times 286.8 \times 1000 \frac{V_y}{2} = 636 \text{ m/s} \]

New \( p = \frac{p R T}{\text{gauge}} \Rightarrow p_2 = p R T_2 \)

From p. 664 of Vennard & Street (U.S. Standard atmosphere):

At altitude of 12 km, \( p = 19.40 \text{ kPa (absolute)} \)

\[ p_2 = 19400 \frac{56.8 \times 1000}{2} \]

\[ m = \frac{p_2 V_y A_2}{\text{gauge}} \]

\[ \text{from (x): } F_{\text{nozzle}} = 90000 = 0.0676 \times 634 \times A_2 (634 - 270) \]

\[ A_2 = 5.77 \text{ m}^2 \]
Solution (0.3)

(a) A streamline through C: No flow can cross a streamline ⇒ A streamline passing through C must pass through B as shown below, such that

\[ \text{Mass through AB} = \text{Mass through DC} \]

At pressure is constant throughout ⇒ \( p = \text{constant} \)

\[ \text{Volume flow through AB} = \text{Volume through DC} \]

for unit depth

\[ \int_0^b u \, dy = \int_0^b u \, dy \quad (\text{dA} = dy \times \text{depth}) \]

or

\[ \int_0^b \frac{u}{U} \, dy \]

As \( u < U \) for \( y < b \), \( h = AB < DC \), as shown.

As the streamline \( L \) is not parallel to the plate, the fluid therefore still has a small \( y \)-component velocity at C.
Applying momentum principles in the x-direction:

\[ \sum F_x = \oint p \nu_x (x \cdot dx) \]

\[ = -F_s \]

Streamline

\[ \oint p \nu_x (x \cdot dx) = p \int_0^\delta \left[ \int_0^\delta u \, dy \right] \]

But from mass conservation:

\[ \rho U h = \int_0^\delta \int_0^\delta u \, dy \]

or

\[ h = \int_0^\delta \frac{\int_0^\delta u \, dy}{U} \]

\[ \therefore -F_s = -\rho U^2 \int_0^\delta \frac{\int_0^\delta u \, dy}{U} + \int_0^\delta u^2 \, dy \]

or

\[ F_s = \rho \int_0^\delta (u - u) \, dy \]

(\[ = \rho U^2 \theta, \]

where \( \theta \) is called

momentum thickness)
(c) air at standard conditions (15°C = 288 K, 101.3 kPa)

\[ p = \frac{P}{RT} = \frac{101300}{287 \times 288} = 1.23 \text{ kg/m}^3 \]

Substituting for \( U = U \left\{ \frac{3}{2} \left( \frac{y}{\delta} \right) - \frac{1}{2} \left( \frac{y}{\delta} \right)^3 \right\} \) in the drag force equation:

\[ D = \int_0^\delta y (U-u) dy = \int_0^\delta y (U-U^2) dy \]

\[ = \int_0^\delta y \left\{ U^2 \left[ \frac{3}{2} \left( \frac{y}{\delta} \right) - \frac{1}{2} \left( \frac{y}{\delta} \right)^3 \right] - U^2 \left[ \frac{3}{2} \left( \frac{y}{\delta} \right) - \frac{1}{2} \left( \frac{y}{\delta} \right)^3 \right]^2 \right\} dy \]

\[ = \int_0^\delta y \left\{ U^2 \frac{3}{2} \left( \frac{y}{\delta} \right) - \frac{1}{2} U^2 \frac{y^3}{\delta^3} - U^2 \frac{3}{4} \frac{y^4}{\delta^4} + U^2 \frac{1}{2} \frac{y^6}{\delta^6} \right\} dy \]

\[ = \int_0^\delta y \left\{ U^2 \left[ \frac{3}{28} \frac{\delta^2}{\delta} - \frac{9}{4} \frac{\delta^3}{\delta^3} - \frac{1}{2} \frac{\delta^4}{\delta^4} + \frac{3}{28} \frac{\delta^5}{\delta^5} - \frac{1}{48} \frac{\delta^7}{\delta^7} \right] \right\} dy \]

\[ = \left( \frac{U^2}{\delta} \left[ \frac{3}{4} - \frac{3}{4} - \frac{1}{8} + \frac{3}{10} - \frac{1}{28} \right] \right) = 0.139 \frac{U^2}{\delta} \]

\[ = 0.139 \times 1.23 \times 10^2 \times 0.0023 = 0.0393 \text{ N per m of plate's depth} \]

(onto the page)