Pipe Flow Example

(Adapted from Street et al., 1995)

Calculate the smallest reliable flowrate that can be pumped through this pipeline. Assume atmospheric pressure 101.3 kPa. Also calculate pump power.

Solution. For reliable flow rate, cavitation must not occur.

As the lowest pressure would occur at A, cavitation appears here first.

Energy eqn A \rightarrow B gives

\[
\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + z_B + f \left( \frac{L}{D} \right) \frac{V^2}{2g}
\]

\[V_A = V_B = V\]

\[P_B = \rho \text{ atm} = 101.3 \text{ kPa}\]

\[P_A = \rho \text{ vapour} = 7.38 \text{ kPa (at } 40^\circ \text{C)}\]

\[z_A - z_B = 15 + 7.5 = 22.5 \text{ m}\]

\[\frac{7380}{1000 \times 9.8} + \frac{V^2}{2g} + 22.5 = \frac{101300}{1000 \times 9.8} + \frac{V^2}{2g} + f \left( \frac{45}{0.025} \right) \frac{V^2}{2g}\]

\[\rightarrow\]
22.5 + \frac{7380}{1000 \times 9.8} - \frac{101300}{1000 \times 9.8} = 12.9 = f(1800) \frac{V^2}{2 \times 9.8}

or \quad 0.140 = f \frac{V^2}{2}

We solve for both V and f iteratively.

For cast-iron pipes, relative roughness \( \frac{a}{d} = \frac{0.26}{25} = 0.0104 \)

From Moody diagram, assume initially \( f = 0.038 \)

\( \Rightarrow \quad \sqrt{\frac{0.140}{f}} = 1.92 \text{ m/s} \)

Re = \frac{f \nu d}{\mu} = \frac{1000 \times V \times 0.025}{0.653 \times 10^{-3}} = 3.83 \times 10^4 \text{ V}

= 7.35 \times 10^4 \text{ m/s}

Moody diagram gives : \( f = 0.0386 \)

\( \Rightarrow \quad \sqrt{\frac{0.140}{f}} = 1.90 \text{ m/s} \)

Since \( V_{\text{new}} \) & the previous \( V \) (1.92 m/s) are pretty close (to within about 5%), we can say the iteration process has converged.

\( \Rightarrow \quad V = 1.90 \text{ m/s} \Rightarrow \text{flow rate} \quad Q = \frac{V \pi d^2}{4} \)

or \( Q = 1.90 \times \pi \times 0.025 \times 0.025 \times 4 = 9.33 \times 10^{-2} \text{ m}^3/\text{s} \)

\( = 0.933 \text{ l/s} \)

To calculate pump power, we apply energy equation from \( 0 \rightarrow A \) (ignoring losses in the short pump inlet pipe):

\( \frac{\rho g}{g} + \frac{V_0^2}{2g} + z_0 + h_{\text{pump}} = \frac{\rho u^2}{2g} + z + h_{\text{friction}} + h_{\text{loss}} - \text{assumed} - \text{assumed} - \text{assumed} \)

\( \Rightarrow \quad 101300 + 0 + h_{\text{pump}} = 0 + \frac{1.90^2}{2 \times 9.8} + 15 + 0.0386(0.025)^2 \quad \text{or} \quad h_{\text{pump}} = 18.5 \text{ m} \Rightarrow \text{Pump power} \quad P = \frac{h_{\text{pump}} ho g V^2}{2} = \frac{18.5 \times 1000 \times 9.8 \times 1.90^2}{2} = 169 \text{ W} \)
9.160 Three pipes join at a common point at elevation 105. One, a 0.3 m line 600 m long, goes to a reservoir of surface elevation 120; another, a 150 mm line 900 m long, goes to a reservoir of surface elevation 150; the third (150 mm) runs 300 m to elevation 75, where it discharges into the atmosphere. Assuming that \( f = 0.020 \), calculate the flowrate in each line. Calculate these flowrates when a 50 mm nozzle is attached to the end of the third pipe.

All pipes are rough with roughness height \( e = 1 \) mm.

Solution:

Assuming flow direction in pipe \#1 as shown. (Note that there are no "choices" regarding flow direction in pipes \#2 \& \#3, as an inspection of the energy equation shown below for these pipes will show.) Ignoring losses at the junction J but taking entrance losses, then

Energy equation given:

\[
A \rightarrow J : \quad \frac{p_1}{ho g} + \frac{v_1^2}{2g} + h_A = \frac{p_f}{ho g} + h_f + \frac{v_2^2}{2g} + h_3 + \frac{1}{2} f \frac{v_2^2}{ho g}
\]

or

\[
120 = \left( \frac{p_f}{ho g} + h_f \right) + \frac{v_2^2}{2g} \left( 1.5 + 2000 f_1 \right)
\]
\[ B \rightarrow J : \quad \frac{p_j}{\rho_j} + \frac{v_j^2}{2g} + \frac{z_j}{g} = \frac{p_j}{\rho_j} + \frac{v_j^2}{2g} + \frac{z_j}{g} + \frac{V_j^2}{2g} + \frac{L_j}{D_j} \frac{v_j}{2g} \]

or \[ 150 = (\frac{p_j}{\rho_j} + \frac{z_j}{g}) + \frac{V_j^2}{2g} \left( 1.5 \times 6000 \frac{f_3}{f_2} \right) \]

\[ J \rightarrow C : \quad \frac{p_j}{\rho_j} + \frac{z_j}{g} + \frac{V_j^2}{2g} = \frac{p_c}{\rho_c} + \frac{V_c^2}{2g} + \frac{z_c}{g} + \frac{L_j}{D_j} \frac{v_j}{2g} \]

or \[ (\frac{p_j}{\rho_j} + \frac{z_j}{g}) = 75 + \frac{V_c^2}{2g} \left( 2000 \frac{f_3}{f_2} \right) \]

Conservation of mass requires: \[ Q_1 + Q_2 = Q_3 \]

or \[ \frac{\pi}{4} D_1^2 v_1 + \frac{\pi}{4} D_2^2 v_2 = \frac{\pi}{4} D_3^2 v_3 \]

\[ Q_1 = \frac{\pi}{4} D_1^2 v_1 = 0.0707 v_1 \]

\[ Q_2 = \frac{\pi}{4} D_2^2 v_2 = 0.0177 v_2 \]

\[ Q_3 = \frac{\pi}{4} D_3^2 v_3 = 0.0177 v_3 \]

Writing \( H_j = \frac{p_j}{\rho_j} + \frac{z_j}{g} \) and simplifying further the above equations, we get

\[ \begin{cases} 
120 = H_j + \frac{V_j^2}{2g} \left( 0.07653 + 102.0 \frac{f_3}{f_1} \right) \\
150 = H_j + \frac{V_j^2}{2g} \left( 0.07653 + 306.1 \frac{f_3}{f_2} \right) \\
H_j = 75 + \frac{V_j^2}{2g} \left( 102.0 \frac{f_3}{f_2} \right) \\
0.0707 v_1 + 0.0177 v_2 = 0.0177 v_3 
\end{cases} \]

To solve this system of equations, we use trial and error method. New \( \frac{e_1}{D_1} = \frac{1}{300} = 0.0033 \)

\[ \frac{e_2}{D_2} = \frac{1}{150} = 0.0067 \] \[ \frac{e_3}{D_3} = \frac{1}{150} = 0.0067 \]

Assuming \( 1 \) range Reynolds numbers \( (Re_1 = Re_2 = Re_3 = 1 \times 10^3) \)
The Moody diagram gives:

\[ f_1 = 0.016 \quad f_2 \approx f_3 = 0.033 \]

Now, assuming \( H_y = 75 \text{ m} \)

\[ V_1 = 5.06 \text{ m/s} \quad Q_1 = 0.287 \text{ m}^3/\text{s} \]
\[ V_2 = 2.71 \quad Q_2 = 0.049 \]
\[ V_3 = 0 \quad Q_3 = 0 \]

\[ V_1 + V_2 + V_3 = 0 \quad \text{Difference} \quad E = V_1 + V_2 - V_3 = 0.335 \quad \text{not OK.} \]

Now try \( H_y = 120 \text{ m} \)

\[ V_1 = 0 \quad Q_1 = 0 \]
\[ V_2 = 1.72 \quad Q_2 = 0.0306 \]
\[ V_3 = 3.66 \quad Q_3 = 0.0647 \]

\[ Q_1 + Q_2 < Q_3 \quad \text{Difference} \quad E = (Q_1 + Q_2) - Q_3 = -0.0343 \quad \text{not OK.} \]

See plot below for an estimated value of \( H_y \) where \( E \) would be near zero. Using the plot, next we try \( H_y = 116 \text{ m} \)

\[ V_1 = 1.21 \quad Q_1 = 0.0856 \]
\[ V_2 = 1.83 \quad Q_2 = 0.0329 \]
\[ V_3 = 3.44 \quad Q_3 = 0.0616 \]

\[ \text{Difference} \quad E = 0.0562 \quad \text{not OK.} \]

Now using the 3 points \((H_y = 70, E = 0.335), (H_y = 116, E = 0.0343)\)

\((H_y = 120, E = 0.0343)\), we estimate that \( E \) would be near zero at \( H_y = 119 \text{ m} \)

\[ \text{Try} \quad H_y = 119 \text{ m} \]

\[ V_1 = 0.605 \quad Q_1 = 0.0429 \]
\[ V_2 = 1.745 \quad Q_2 = 0.0309 \]
\[ V_3 = 3.616 \quad Q_3 = 0.0660 \]

\[ E = (Q_1 + Q_2) - Q_3 = +0.0097 \quad \text{still not OK.} \]

Now try \( H_y = 119.6 \text{ m} \)

\[ V_1 = 0.469 \text{ m/s} \quad Q_1 = 0.0332 \text{ m}^3/\text{s} \]
\[ V_2 = 1.734 \quad Q_2 = 0.0307 \]
\[ V_3 = 3.632 \quad Q_3 = 0.0643 \]

\[ E = -0.0004 \quad \text{or} \quad E/Q_3 \approx 0.06 \quad \text{close enough?} \quad \text{OK.} \]
\[ E = (Q_1 + Q_2) \cdot Q_3 \]

Graph with axes labeled \( E \) and \( H_x \) and a curve plotted on the graph paper.
Now, check the f's values.

\[ V_1 = 0.669 \]  \( \Rightarrow \) \( f_1 = \frac{V_1 D_1}{\mu} = \frac{1000 \times 0.669 \times 0.3}{1 \times 10^{-3}} = 1.4 \times 10^5 \) 

\( (e/D)_1 = 0.0033 \) \( \Rightarrow \) \( f_1 = 0.0275 \) \( \ldots \) \( \) not ok.

\[ V_2 = 1.73 m \]  \( \Rightarrow \) \( f_2 = \frac{1000 \times 1.734 \times 0.15}{1 \times 10^{-3}} = 2.6 \times 10^5 \)

\( (e/D)_2 = 0.0067 \) \( \Rightarrow \) \( f_2 = 0.033 \) \( \) ok.

\[ V_3 = 3.632 \]  \( \Rightarrow \) \( f_3 = \frac{1000 \times 3.632 \times 3.632 \times 0.15}{1 \times 10^{-3}} = 5.4 \times 10^5 \)

\( (e/D)_3 = 0.0067 \) \( \Rightarrow \) \( f_3 = 0.033 \) \( \) ok.

As \( f_1 \) has been wrongly assumed, we need to iterate again.

Now, assume \( f_1 = 0.0275 \) \( \); \( f_2 = f_3 = 0.033 \).

Then, try \( H_2 = 119.4\) m

\( \Rightarrow \) \( V_1 = 0.456 \); \( Q_1 = 0.0323 \)

\( V_2 = 1.734 \); \( Q_2 = 0.0302 \)

\( V_3 = 3.632 \); \( Q_3 = 0.0663 \)

\( E = 0.0013 \) ; \( E/Q_3 = 2\% \)

As the error \( E \) is fairly small (less than 5\%),
we accept these \( Q's \) values as approximately correct if the \( f's \) values are also correct.

Now, from above, \( f_2 \) \& \( f_3 \) are correct (\( = 0.033 \))
As for \( f_1 \), \( V_1 = 0.456 \) m/s \( \Rightarrow \) \( R_1 = 1.4 \times 10^5 \)

with \( (e/D)_1 = 0.0033 \) \( \Rightarrow \) \( f_1 = 0.0275 \) from Henry's diagram \( \Rightarrow \) \( f_1 \) is also correct.

\[ Q_1 = 0.0323 \text{ m}^3/\text{s} \] \( \) in the direction as shown (toward the junction)

\[ Q_2 = 0.0707 \text{ m}^3/\text{s} \]

\[ Q_3 = 0.0663 \text{ m}^3/\text{s} \]
with a 50 mm nozzle fitted to end of pipe #3.

Energy equation \( J \rightarrow c \) now becomes:

\[
\left( \frac{P_2}{\rho g} + g_f \right) + \frac{V_2^2}{2g} = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + e_1 + fL \frac{V_1^2}{2g}
\]

\[\text{New} \quad V_3A_3 = V_{nose}A_{nose} \Rightarrow \frac{V_3}{\sqrt{2}} (0.15)^2 V_3 = \frac{V_{nose}}{\sqrt{2}} (0.050)^2 V_{nose}\]

\[V_{nose} = 9 V_3\]

\[
\left( \frac{P_2}{\rho g} + g_f \right) = \frac{V_2^2}{2g} (9^2 - 1 + 2000 f_3)
\]

\[H_3 = 75 + V_3^2 (4.0^2 + 10^2 f_3)\]

The system of equations becomes:

\[
\begin{align*}
120 & = H_3 + V_3^2 (0.0365^2 + 102.0 f_3) \\
150 & = H_3 + V_3^2 (0.0765^2 + 106.1 f_3) \\
H_3 & = 75 + V_3^2 (4.0^2 + 10^2 f_3) \\
0.037V_1 + 0.017V_2 & = 0.0172 V_3
\end{align*}
\]

Assume \( f_1 = 0.0375 \); \( f_3 = 0.033 \) (from above)

Try \( H_3 = 120 \text{ m} \)

\[\begin{align*}
V_1 &= 0 \quad ; \quad Q_1 = 0 \\
V_2 &= 1.72 \quad ; \quad Q_2 = 0.0306 \\
V_3 &= 2.46 \quad ; \quad Q_3 = 0.0435 \\
E &= (Q_1 + Q_2) - Q_3 = -0.0131 \text{ m}^3/s, \quad \text{not ok.}
\end{align*}\]

Try \( H_3 = 119.4 \text{ m} \)

\[\begin{align*}
V_1 &= 0.456 \quad ; \quad Q_1 = 0.0323 \\
V_2 &= 1.734 \quad ; \quad Q_2 = 0.0307 \\
V_3 &= 2.442 \quad ; \quad Q_3 = 0.0432 \\
E &= +0.0198 \quad \text{not ok.}
\end{align*}\]
Try \( H_J = 119.7 \text{ m} \)

\[ v_1 = 0.323 \text{ m/s} \quad q_1 = 0.0222 \text{ m}^3/\text{s} \]
\[ v_2 = 1.725 \quad q_2 = 0.0305 \]
\[ v_3 = 2.450 \quad q_3 = 0.0434 \]
\[ E = 0.0099 \quad \text{not ok} \]

Try \( H_J = 119.2 \text{ m} \)

\[ v_1 = 0.263 \quad q_1 = 0.0186 \]
\[ v_2 = 1.723 \quad q_2 = 0.0305 \]
\[ v_3 = 2.453 \quad q_3 = 0.0434 \]
\[ E = 0.0057 \quad \text{not ok} \]

Try \( H_J = 119.9 \text{ m} \)

\[ v_1 = 0.186 \text{ m/s} \quad q_1 = 0.0132 \text{ m}^3/\text{s} \]
\[ v_2 = 1.720 \quad q_2 = 0.0304 \]
\[ v_3 = 2.456 \quad q_3 = 0.0435 \]
\[ E = 1 \times 10^{-4} \quad E/q_3 = 0.2\% \quad \text{close enough to ok} \]

New check for \( f_1 \):
\[ f_1 = 0.033 \quad q_1 = 0.186 \text{ m/s} \quad Re_1 = 5.6 \times 10^4 \quad f_1 = 0.029 \quad \text{not ok} \]
\[ v_2 = 1.720 \Rightarrow Re_2 = 2.6 \times 10^5 \quad \left( \frac{E}{q_3} \right) = 0.067 \Rightarrow f_2 = 0.038 \quad \text{ok} \]
\[ v_3 = 2.456 \Rightarrow Re_3 = 1.4 \times 10^5 \quad \left( \frac{E}{q_3} \right) = 0.068 \Rightarrow f_3 = 0.033 \quad \text{ok} \]

New Assume \( f_1 = 0.029 \); \( f_2 = f_3 = 0.033 \)

Try \( H_J = 119.9 \text{ m} \)

\[ v_1 = 0.182 \text{ m/s} \quad q_1 = 0.0129 \text{ m}^3/\text{s} \]
\[ v_2 = 1.720 \quad q_2 = 0.0304 \]
\[ v_3 = 2.456 \quad q_3 = 0.0435 \]
\[ E = 2 \times 10^{-4} \quad E/q_3 = 0.5\% \quad \text{close enough to ok} \]

Check for \( f_1 \):
\[ Re_1 = 5.5 \times 10^4 \Rightarrow f_1 = 0.029 \quad \text{ok} \]

Also \( Re_2 = 2.6 \times 10^5 \Rightarrow f_2 = 0.038 \); \( Re_3 = 1.4 \times 10^5 \Rightarrow f_3 = 0.033 \) \( \text{ok} \)

\[ \text{Answers:} \quad q_1 = 0.0129 \text{ m}^3/\text{s}, \quad q_2 = 0.0304 \text{ m}^3/\text{s}, \quad q_3 = 0.0435 \text{ m}^3/\text{s} \]
Note that had we assumed the flow direction in pipe 31 to be from the junction to the reservoir, then the corresponding Bernoulli equiv. becomes:

\[ \frac{P_0}{ho} + \frac{V_1^2}{2g} + z_1 = \frac{P_0}{ho} + \frac{V_2^2}{2g} + z_2 + \frac{f_1 L_1 V_1^2}{2g} + \frac{f_2 L_2 V_2^2}{2g} \]

or \( \frac{P_0}{ho} + \frac{V_1^2}{2g} + z_1 = \frac{P_0}{ho} + \frac{V_2^2}{2g} + z_2 + \frac{f_1 L_1 V_1^2}{2g} + \frac{f_2 L_2 V_2^2}{2g} \)  

\[ \text{exit loss} \]

and the first system (without nozzles) becomes:

\[
\begin{align*}
H_J &= 120 + (102 f_1) V_1^2 \\
150 &= H_J + (0.02653 + 3 \times 6.1 f_2) V_2^2 \\
H_J &= 75 + (102 f_3) V_3^2 \\
0.0787 V_1 + 0.0177 V_2 &= 0.0177 V_3
\end{align*}
\]

An inspection of these equations shows that:

\[ H_J \geq 75, \quad H_J \geq 120 \quad \text{(as} \ V_1^2 \geq 0) \]

\[ H_J \leq 150 \quad \text{(as} \ V_2^2 \geq 0) \]

i.e., \[ 120 \leq H_J \leq 150 \]

Now, assume \( f_1 = 0.0275, \quad f_2 = f_3 = 0.033 \).

Try \( H_J = 120 \) m

\[ V_1 = 0 \quad \Rightarrow \quad Q_1 = 0 \]
\[ V_2 = 1.22 \quad \Rightarrow \quad Q_2 = 0.0364 \]
\[ V_3 = 3.66 \quad \Rightarrow \quad Q_3 = 0.0647 \]

Error \( E = Q_2 - (Q_2 + Q_1) = 0.0343 < 0 \)

Next try \( H_J = 150 \) m

\[ V_1 = 3.22 \quad \Rightarrow \quad Q_1 = 0.231 \]
\[ V_2 = 0 \quad \Rightarrow \quad Q_2 = 0 \]
\[ V_3 = 4.32 \quad \Rightarrow \quad Q_3 = 0.0835 \]

Error \( E = -0.0315 < 0 \)

\[ \text{The Error E does not cross the "zero line" thus solution possible.} \]
Fluid Mechanics - Tut. Problems
(from Vennard Street) Example
Man. 28/7/2001

Water is flowing. For $Q_3 = 110$ l/s, calculate $Q_1$, $Q_2$, and pump power.

Problem 9.163

Solution: Flow directions are assumed as shown.

$$Q_3 = 0.110 \text{ m}^3\text{s}^{-1} \Rightarrow V_3 = \frac{Q_3}{A_3} = \frac{0.110}{50/4} = 0.279 \text{ m/s}$$

Friction head loss in pipe 2:

$$h_2 = \frac{L_2}{L} \left( \frac{g}{D_2} \right) V_2^2 \Rightarrow h_2 = \frac{0.279}{2} \left( \frac{g}{D_2} \right) V_2^2$$

Applying Energy Eqn between $\mathbb{B} \rightarrow \mathbb{A}$, $\mathbb{B} \rightarrow \mathbb{C}$ yields:

$$\frac{V_2^2}{2g} = \frac{90 + h_3 + \frac{V_2^2}{2g}}{2g} \Rightarrow V_2 = 1.55 \text{ m/s}$$

or.

$$30 - h_3 - h_2 = 30 - 18.7 = h_2$$

Therefore, the assumed direction of flow in pipe 2 should be reversed.

$$h_2 = \frac{L_2}{2} \left( \frac{g}{D_2} \right) V_2^2 = 0.023 \left( \frac{600 \text{ m}^3\text{s}^{-1}}{0.15 \text{ m}^3\text{s}^{-1}} \right) \frac{1.55^2}{2g} \Rightarrow V_2 = 1.55 \text{ m/s}$$

Calculating $Q_1$, $Q_2$, and pump power:

$$Q_1 = Q_3 - Q_2 = 0.274 = 0.0826 \text{ m}^3\text{s}^{-1}$$

$$V_1 = \sqrt{\frac{Q_1}{A_1}} = \sqrt{\frac{0.274}{50/4}} = 1.17 \text{ m/s}$$

$$h_1 = h - \frac{V_1^2}{2g} = 1.9 \text{ m} \Rightarrow E_{10} = \left( \frac{V_1^2}{2g} + \frac{1}{2} \right)$$

$$h_2 = 0.018 \left( \frac{0.3}{0.15} \right) \frac{1.7^2}{2g} = 0.9 \text{ m} \Rightarrow E_{10} = \left( \frac{V_1^2}{2g} + \frac{1}{2} \right)$$
To obtain \( \left( \frac{P_p}{q} + \frac{3P}{q} \right)_i \), we apply energy eqn between \( A \rightarrow \text{P}_i \)

\[ \Delta z = \left( \frac{P_p}{q} + \frac{3P}{q} \right)_i + h_L \]

or \( \left( \frac{P_p}{q} + \frac{3P}{q} \right)_i = \Delta z - 1.9 = 28.1 \text{ m} \)

\[ \text{Pump head} \quad E_p = \left( \frac{P_p}{q} + \frac{3P}{q} \right)_i - \left( \frac{P_p}{q} + \frac{3P}{q} \right)_o \]

\[ = (90 - 11.3) - (28.1) \]

\[ = (60 + 18.7) \text{ from above} \]

\[ E_p = (60 + 18.7) = 58.6 \text{ m} \]

\[ \text{Power} \quad P = \rho g Q_i E_p = 9.8 \times 1000 \times 0.0926 \times 58.6 / 1000 \]

\[ P = 4.1 \text{ kW} \]

(Note: only \( Q_i \) is effectively flowing through the pump)

Energy lines are as shown above.
**Problem.**

Example - Pipe Flow

Find flow rate for the following situation:

**Solution.**

Writing the energy equation between the reservoir free surface (0) and the pipes exit (1) yields:

\[
\frac{h_0}{g} + \frac{\dot{V}_0^2}{2g} + z_0 = \frac{h_1}{g} + \frac{\dot{V}_1^2}{2g} + z_1 + h_l
\]

\[3_0 - 3_1 = 10\ m, \ \text{thus} \]

\[\frac{\dot{V}_1^2}{2g} + h_l = 10 \ \text{(here} \ V_1 = \text{Vel. in the 3cm pipe)}
\]

Now \[h_l = h_{\text{entrance}} + h_{\text{connection}} + h_{\text{valve}} + h_{\text{elbow}} + h_{\text{friction in 3cm pipe}} + h_{\text{friction in 3cm pipe}}
\]

Now, let \(V_5\) = Velocity in 5 cm pipe

\[V_3 = " " \ 3 \ cm " \]

Then \[V_5 A_5 = V_3 A_3 \text{ or } \frac{V_5}{V_3} = \frac{A_5}{A_3} = \frac{\pi d_5^2/4}{\pi d_3^2/4} = \left(\frac{d_5}{d_3}\right)^2 = \left(\frac{5}{3}\right)^2
\]

or \[V_5 = 0.36 V_3 \quad = 2.778
\]
Now \( l_{\text{inflow}} = k_{\text{in}} V_{5/2g} = 0.5 \cdot V_{5/2g} \) (p. 392)

\[ l_{\text{connection}} = k_{\text{con}} (V_{5} - V_{3})/2g \]

\[ = 0.326 (V_{5} - V_{3})/2g \quad \Rightarrow \quad V_{5} = 0.326 V_{3} \] (p. 392)

\[ l_{\text{value}} = 10 V_{3/2g} \] (p. 392)

\[ l_{\text{allow}} = 1.5 V_{3/2g} \] (p. 392)

\[ (l_{f}) = \frac{V_{5}^{2}}{d} \cdot \frac{V_{5}}{2g} \quad \Rightarrow \quad (\frac{V_{5}}{d}) = \frac{0.0009}{10} \] (p. 382)

\[ (l_{f}) = \frac{V_{5}^{2}}{d} \cdot \frac{V_{5}}{2g} = \frac{V_{5}^{2}}{d} \cdot \frac{0.0009}{10} = 5.1 \frac{V_{5}^{2}}{d} \]

\[ (l_{f})_{\text{friction}} = \frac{V_{5}^{2}}{d} \cdot \frac{V_{5}}{2g} = \frac{V_{5}^{2}}{d} \cdot \frac{0.0009}{10} = 666.7 \frac{V_{5}^{2}}{d} \]

\[ 10 = \frac{V_{5}^{2}}{2g} + \left[ \frac{0.065 + 0.133 + 10 + 1.5 + 51.8}{f_{3}} + 666.7 \frac{V_{5}^{2}}{d} \right] \frac{V_{5}^{2}}{2g} \]

\[ \Rightarrow \quad 10 = \left( 12.70 + 51.8 \frac{V_{5}}{d} + 666.7 \frac{V_{5}^{2}}{d} \right) \frac{V_{5}^{2}}{2g} \]

\[ \Rightarrow \quad (\frac{V_{5}}{d}) = 0.0015 \] (p. 372)

\[ V_{5} = \left[ \frac{10 \times 5.2 \times 9.8}{12.70 + 51.8 \times 0.019 + 666.7 \times 0.021} \right]^{1/2} = \left[ \frac{10 \times 5.2 \times 9.8}{12.70 + 51.8 \times 0.019 + 666.7 \times 0.021} \right]^{1/2} \]

\[ V_{5} = 2.66 \text{ miles} \]

\[ \Rightarrow \quad R_{3} = \left( \frac{V_{5}}{d} \right) = \frac{2.66 \times 0.03}{1.14 \times 10^{-6}} = 7 \times 10^{4} \]

\[ (\frac{V_{5}}{d}) = 0.0015 \]

\[ V_{5} = 0.36 V_{3} = 0.96 \text{ miles} \]

\[ \Rightarrow \quad R_{5} = \left( \frac{V_{5}}{d} \right) = \frac{0.96 \times 0.03}{1.14 \times 10^{-6}} = 4.2 \times 10^{4} \quad \Rightarrow \quad (\frac{V_{5}}{d}) = 0.0045 \]

\[ (\frac{V_{5}}{d}) = 0.0009 \]

\[ (\frac{V_{5}}{d}) = 0.019 \]
Try:

\[ f_3 = 0.023 \] \[ f_5 = 0.022 \]

\[ V_3 = \sqrt{\frac{10 \times 2 \times 9.8}{12.7 + 51.8 \frac{f_5}{2}} + 666.7 \frac{f_3}{5}} = \sqrt{\frac{10 \times 2 \times 9.8}{12.7 + 1.14 + 15.33}} \]

= 2.6 m/s

\[ k_3 = \frac{\frac{V_3}{D}}{3} = \left( \frac{2.6 \times 0.03}{1.14 \times 10^{-6}} \right) = 6.8 \times 10^4 \Rightarrow f_3 = 0.0245 \]

\( \left( \frac{E}{d} \right) \frac{f_3}{3} = 0.0015 \)

\[ k_5 = \frac{\frac{V_5}{D}}{5} = \left( \frac{2.6 \times 0.36 \times 0.05}{1.14 \times 10^{-6}} \right) = 4.1 \times 10^4 \Rightarrow f_5 \leq 0.0245 \]

\( \left( \frac{E}{d} \right) \frac{f_5}{5} = 0.0009 \)

Try:

\[ f_3 = 0.024 \] \[ f_5 = 0.024 \]

\[ V_3 = \sqrt{\frac{10 \times 2 \times 9.8}{12.7 + 1.243 + 16.001}} = 2.56 m/s \]

\[ k_3 = \frac{2.56 \times 0.03}{1.14 \times 10^{-6}} = 6.7 \times 10^4 \Rightarrow f_3 = 0.0245 \]

\[ \left( \frac{E}{d} \right) \frac{f_3}{3} = 0.0015 \] Close

\[ k_5 = \frac{(2.56 \times 0.36 \times 0.05)}{1.14 \times 10^{-6}} = 4.04 \times 10^4 \Rightarrow f_5 \leq 0.0245 \]

\( \left( \frac{E}{d} \right) \frac{f_5}{5} = 0.0009 \) Close

\[ Q = V_3 A_3 = \frac{\pi}{4} (0.03)^2 \times 2.58 = 0.0018 m^3/s \]