Example (Ref. Fay)

Application of Bernoulli: 

Falling fluid jet

Applying Bernoulli's between

\[ \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \phi_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \phi_2 \quad (a) \]

\[ P_1 \text{ and } P_2 \text{ are equal to the} \]

surrounding atmosphere:

From statics:

\[ P_2 - P_1 = \rho [\phi_2 - \phi_1] \quad (b) \]

From (c):

\[ P_2 - P_1 = \ldots \]

\[ V_2^2 = V_1^2 + 2g(\phi_2 - \phi_1) \]

\[ \frac{\rho_1}{\rho_w} = 10^{-3} \ll 1 \]

\[ V_2^2 = V_1^2 + 2g(\phi_2 - \phi_1) \]

Also

\[ \rho V_2 \frac{\pi}{4} d_2^2 = \rho V_1 \frac{\pi}{4} d_1^2 \]

\[ d_2 = \sqrt{\frac{\rho V_1^2}{V_1^2 + 2g(\phi_2 - \phi_1)}} \]

\[ d_2 \text{ decreases} \]
3. A 0.3 m by 0.5 m rectangular air duct carries a flow of 0.45 m³/s at a density of 2 kg/m³. Calculate the mean velocity in the duct. If the duct tapers to 0.15 m by 0.5 m size, what is the mean velocity in this section if the density is 1.5 kg/m³ there?

(Ans. 8 m/s)

4. Find \( \dot{V} \) for this mushroom cap on a pipeline.

(Ans. 1.8 m/s)

5. (From Gerhart et al.)

(a). Air enters the axial compressor shown in Fig. P4.15 at 101 kPa absolute, 27°C, and an average axial velocity component of 1.5 m/s. Find the air mass flow rate through the compressor.

(b). The absolute pressure and temperature at point 2 of the axial compressor in Fig. P4.15 are 202 kPa and 87°C. The air mass flow rate is 5.33 kg/s. Calculate the average axial velocity at point 2.

(Ans. 5.53 kg/s, 1.20 m/s)

6. Show that the flow field described by the velocity components

\[ u = \frac{-2xye}{(x^2 + y^2)^3}, \quad v = \frac{(x^2 - y^2)e}{(x^2 + y^2)^3}, \]

and

\[ w = \frac{y}{x^2 + y^2} \]

is a possible incompressible fluid flow.
(3) 

\[ V = \frac{Q}{A} = \frac{0.45}{0.3 \times 0.5} = 3 \text{ m/s} \]

Mass flow rate \( m = (\dot{m}) = PQ \)

Mass conservation gives:

\[ (PQ)_1 = (PQ)_2 \quad \text{or} \quad (PVA)_1 = (PVA)_2 \]

\[ V_2 = \frac{(PQ)_1}{(P/A)_2} = \frac{2.0 \times 0.45}{1.5 \times 0.15 \times 0.5} = 4 \text{ m/s} \]
Conservation of mass requires:

\[ Q_{in} = Q_{out} \]

or

\[ 3 \text{ m}^3/\text{s} = V \cdot A \]

\[ = V \cdot \cos 45^\circ \cdot \left( \frac{\pi \cdot 2^2}{\pi \cdot 1.8^2} \right) \]

\[ V = 1.78 \text{ m/s} \]
PROBLEM 4.15

GIVEN: Ideal gas at 101 kPa absolute and 27°C and average axial velocity component of 1.5 m/s in 2.0 m circular inlet. Figure P4.15.

FIND: Air mass flow rate.

Solution: Using the ideal gas law and Table A.3, the density is

\[ \rho = \frac{P}{RT} = \frac{101 \text{ kPa}}{(2.77 \text{ kPa/kg-K})(273+27) \text{ K}} = 1.17 \text{ kg/m}^3 \]

The mass flow rate is

\[ \dot{m} = \rho A \dot{V} = \rho (\pi d^2/4) \dot{V} \]

\[ \dot{m} = \rho (\pi (1.5 \text{ m})^2) \cdot 5 \text{ m/s} = 5.53 \text{ kg/s} \]

(DNW)
PROBLEM 4.16

GIVEN: Fig. P4.15 with absolute pressure 202 kPa and temperature 87°C at point 2. Air mass flow rate is 5.53 kg/s.

Find Average axial velocity at section 2.

Solution: Assuming air is an ideal gas, the ideal gas law and Table A.3 gives:

\[ P = \frac{RT}{\gamma} \left( \frac{287.0 \text{ N} \cdot \text{m}^2 / \text{kg} \cdot \text{K}}{273 + 87} \right) \]

The average axial velocity is

\[ \bar{V} = \frac{\dot{m}}{FA} = \frac{4\Delta \rho}{\pi F (d_0^2 - d_1^2)} \]

\[ \bar{V} = 1.20 \text{ m/s} \]
(6) For a possible incompressible fluid flow, one needs

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]

Now, for this problem:

\[ \frac{\partial u}{\partial x} = \frac{(x^2 + y^2)^2(-2y) + 2xyz(2)(2x)(x^2 + y^2)}{(x^2 + y^2)^4} \]

\[ \frac{\partial v}{\partial y} = \frac{(x^2 + y^2)^2[-2yz] - (x^2 - y^2)z(2)(2y)(x^2 + y^2)}{(x^2 + y^2)^4} \]

\[ \frac{\partial w}{\partial z} = 0. \]

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{-2yz(x^2 + y^2) + 2x^2yz(x^2 + y^2) - 2yz(x^2 + y^2)}{(x^2 + y^2)^4} \]

\[ = 0 \]

\[ \therefore \text{Flow is possible.} \]
A long solid circular cylinder of radius R spins steadily about its own axis in a stream of fluid of density \( \rho \), as shown. Far upstream of the cylinder, the fluid has a uniform velocity \( U \) in the positive \( x \) direction, and pressure \( p_0 \). Assume that the flow conditions are such that the fluid velocity on the cylinder surface is given by

\[
V_s = -(2U \sin \theta + K)
\]

where \( K \) is a positive constant relating to rotational speed of the cylinder.

(a) Determine the pressure at points A, B, and C on the cylinder surface in terms of \( p_0 \), \( U \), \( \rho \), \( \theta \), and \( K \). Assume ideal flow and ignore the effects of gravity.

(b) Show that the total force in the \( y \) direction (the lift force) on the cylinder (per unit length) is

\[
L = \rho U (2\pi R K)
\]

Notes: If \( I = \int_0^{2\pi} (\sin^n \theta) \, d\theta \),

then \( I = 0 \) when \( n \) is odd, and \( I = \pi \) when \( n = 2 \).
\[ \rho A^2 \theta = \frac{\pi}{2} : \quad V_S = -(2U + K) \]

Bernoulli \( O \rightarrow A \):

\[ \frac{\rho}{\gamma} + \frac{U^2}{2} = \frac{P_A}{\gamma} + \frac{V_A^2}{2} \quad \Rightarrow \quad \frac{P_A}{\gamma} + \frac{(2U + K)^2}{2} \]

\[ \rho_A = \left[ \frac{\rho}{\gamma} + \frac{U^2}{2} - \frac{(2U + K)^2}{2} \right] \gamma \]

\[ \rho A^2 \theta = -\frac{\pi}{2} : \quad V_S = -(K - 2U) \]

\[ \rho_B = \left[ \frac{\rho}{\gamma} + \frac{U^2}{2} - \frac{(K - 2U)^2}{2} \right] \gamma \Rightarrow \rho_B > \rho_A \]

\[ \rho A C : \quad V_1 = -(2U \sin \theta + K) \]

\[ \rho_C = \left[ \frac{\rho}{\gamma} + \frac{U^2}{2} - \frac{(2U \sin \theta + K)^2}{2} \right] \gamma \]

\[ \text{Force on cylinder (fr. net. flow)}: \]

\[ \text{Net Force} = \int (P_B \cdot R) \cdot \sin \theta \quad \theta = 0 \]
\[
\text{Force} = \int_0^{\pi R} \left( R \cdot \sin^2 \left[ \left( \frac{\rho_0}{l} + \frac{U^2}{l^2} \right) \right] + \frac{(2 \pi \sin \theta + \kappa)^2}{\gamma} \right) \, d\theta
\]
\[
= R \left[ \int_0^{\pi R} \left( \frac{\rho_0}{l} + \frac{U^2}{l^2} \right) \sin \theta \, d\theta - \frac{\sin \theta}{\gamma} \left( 2 \pi \sin \theta + \kappa \right) \right] \, d\theta
\]
\[
\approx \int_0^{\pi R} \sin \theta \, d\theta = \pi R \quad \text{for odd } R
\]

\[
\text{Lift Force} = \frac{R}{\gamma} \int_0^{\pi R} 4U \kappa \sin^2 \theta \, d\theta
\]
\[
= \frac{R}{\gamma} \cdot 4U \kappa \int_0^{\pi R} \sin \theta \, d\theta = \frac{\pi R}{\gamma} \cdot 4U \kappa
\]
\[
= \pi R^2 U \kappa = \rho U (2 \pi R \kappa) \quad (8)
\]