Examples

2. Calculate the moment about $O$ of the resultant force exerted by the water on this half cylinder, which is 3 m long.

(Vennard & Street)

$$M = 22.7 \text{ kN}\cdot\text{m}$$

3. If this solid concrete (24 kN/m$^3$) overhang $ABCD$ is added to the dam, what additional force (magnitude and direction) will be exerted on the dam?

(Vennard & Street)

$$F_{\text{added}} = 3270 \text{ kN}$$

4. This weightless spherical shell with attached small piezometer tube is suspended by a cable as shown. Calculate the total tension force in the cable. Also calculate the force by the liquids: (a) on the bottom half of the sphere, and (b) on the top half of the sphere.

(Vennard & Street)

$$F_{\text{cable}} = 22.43 \text{ kN}$$

$$F_{\text{bottom}} = 37.15 \text{ kN}$$

$$F_{\text{top}} = 14.72 \text{ kN}$$
Horizontal Forces cancelled out

\( F_{\text{ho}} \) cancel \( F_{\text{ac}} \) in \( AC \).

**Vertical Components**

\[ F_{vA} = \text{Weight of enclosed volume} \]
\[ = 1000 \times 9.8 \times \frac{\pi}{2} \times 1.05^2 \times 3 \]
\[ = 50900 \text{ N} \]

This force acts through centroid of \( AOBCA \), which is at \( \frac{4L}{5} = \frac{4 \times 1.05}{3 \times 0.6} = 0.446 \text{ m} \) from \( AB \).

Moment about \( O = M_0 = F_{vA} \times 0.446 \]
\[ = 50900 \times 0.446 \]
\[ = 22680 \text{ N}\cdot\text{m} \]
\[ = 22.7 \text{ kN}\cdot\text{m} \]

Note that \( F_{\text{vert}} \) can be seen to be the resultant of the two vertical forces acting on \( AC \) & \( BC \) separately.

Vertical force on \( AC \) is equal to weight of fluid in \( ACDE \) & acts through its centroid. Similarly, vertical force on \( BC \) is equal to weight of fluid in \( BCDE \) & acts upwards through its centroid.
Given: Overhang in concrete block DDEA is added.

Find: Additional force (magnitude & direction) on dam.

Solution: Treat DCE as a single curved surface.

Weight of Concrete
\[ W = \left(24 \text{ KN/m}^3\right) \cdot \text{Vol. of DDEA} \]

\[ = 24 \times \left(\text{Vol. ABOD} - \text{Vol. DOC}\right) = 24 \times \left(30 \times 3 \times 4.5 - \frac{1}{4} \pi \times 3^2 \right) \]

\[ = 4,630 \text{ KN} \]

Additional horizontal force = \( \frac{\text{Additional projected area}}{\text{Area projected area}} \)

Vertical force due to fluid = \( \theta \times \text{Weight of fluid in DCEF} \)

\[ = 9,800 \left(3 \times 3.9 - \frac{1}{4} \pi \times 3^2 \right) = 136 \times 10^3 \text{ KN} \]

\[ = 1360 \text{ KN} \]

Net vertical force added:
\[ = 4630 - 1360 = 3270 \text{ KN} \]

(If you are interested, the horizontal force on the added concrete overhang is:
\[ F_h = \theta \times (\text{Projected area of ECD}) = 9,800 \times \frac{1}{2} (3.9 \times 3.9) \]

\[ = 2,240 \text{ KN} \]
\[ p_{\text{water}} = 1000 \text{ kg/m}^3; \quad p_{\text{CCL}_4} = 1590 \text{ kg/m}^3. \]

Force in cable \( F_{\text{cable}} = \text{weight of water} + \text{weight of CCL}_4 \)

\[
F_{\text{cable}} = (p_{\text{water}}g + p_{\text{CCL}_4}g) \frac{4}{3} \pi R^3 = (1000 + 1590)(9.8) \frac{4}{3} \pi \times 0.75^3
\]

\[ = 22430 \text{ N} = 22.43 \text{ KN} \]

Force on bottom half ABC:

Total \( F_{\text{bottom}} = \text{weights of liquids above bottom half ABC} \)

\[ = \text{weight of water in ACEF} + \text{weight of CCL}_4 \text{ in ABC} \]

\[ = 1000 \times 9.8 \times (\pi \times 0.75^2) \times (0.75 + 0.6) + 1590 \times 9.8 \times \frac{4}{3} \pi \times 0.75^3
\]

\[ = 37150 \text{ N} = 37.15 \text{ KN} \]

Force on top half ADC:

\[ F_{\text{top}} = \text{weight of water above ADC} \]

\[ = p_{\text{water}} g \times \text{volume ADECF} \]

\[ = 1000 \times 9.8 \times [\pi \times 0.75^2 (0.75 + 0.6) - \frac{4}{3} \pi \times 0.75^3] \]

\[ = 14720 \text{ N} = 14.72 \text{ KN} \]

Note: \( F_{\text{cable}} = F_{\text{bottom}} - F_{\text{top}} \)
Determine the difference in water level between the two tanks shown in the figure below.

\[ Q_1 \]

\[ p_E = p_F \]

\[ \rho = p_E = \rho_f \cdot g \cdot \frac{FE}{g} + \rho_{water} g (x + y) \]

\[ \rho = p_A = \rho_e + \rho_{water} g (0.45 + x) \]

\[ \rho + \rho_{water} g (0.45 + x) = \rho + \rho_{water} g (0.45 + x) \]

\[ \rho = 850 \times 0.45 + 1000 \times 0.45 \]

\[ \rho = 0.45 \times 1000 + 1000 \times 0.45 \]

\[ y = 850 \times 0.45 + 1000 \times 0.45 \]

\[ y = 0.0675 \text{ m} = 67.5 \text{ mm} \]

\[ Q_2 \]
In the dry adiabatic model of the atmosphere, in addition to the state equation for a perfect gas, air is also assumed to obey the equation

\[ \frac{p}{\rho^n} = \text{constant} \]

where \( n = 1.4 \). If the conditions at sea level are standard (101.3 kPa, 15°C), determine the pressure and temperature at an altitude of 4000 m above sea level.
\[ \frac{dp}{dz} = -pg \]

\[ dp/p = -g dz \]

New: \( P/P^n = \text{constant} \quad C \Rightarrow \quad P = (p/c)^{1/n} \)

\[ C = (p/c) \quad \text{sea level} \]

At sea level: \( P/p = RT \Rightarrow \quad P = p/RT; \quad K = 286.8 \quad \text{for air} \)

\[ \frac{P}{p} = \frac{101300}{286.8 \times (673 + 15)} \]

\[ = 1.226 \quad \text{kg/m}^3 \]

\[ C = \frac{101300}{1.226} = 81600 \]

\[ p = \left( \frac{1}{81600} \right)^{1/1.4} = 0.7143 \]

\[ p = \frac{P}{3069} = 0.0003258 \quad p \cdot 0.7143 \]

\[ dp = 0.0003258 \cdot 0.7143 \cdot g \]

Or:

\[ (313.2) \quad \frac{dp}{p \cdot 0.7143} = -dz \]

\[ 313.2 \int \frac{dp}{p \cdot 0.7143} = -\int dz \]

\[ 313.2 \left[ \log p \right]_{0}^{z_0} = -z_{0}^{z_0} \]

\[ 313.2 \left[ \frac{p - P_{sea}}{P_{sea}} \right] = -z_{0}^{z_0} \]

\[ 313.2 \left[ \frac{P_{sea} - 101300}{101300 - 0.2857} \right] = -4000 + 0 \]

\[ P_{sea} = \left[ -4000 \times 0.2857/313.2 + 101300 \right]^{1/0.2857} \]

\[ P_{sea} = 60240 \quad \text{Pa} \]

\[ \theta_{d_000} = \left( P/PR \right)_{d_000} \]
but \[ P_{4000 \text{m}} = \left( \frac{P_{1000 \text{m}}}{C} \right)^{\frac{1}{4}} \]

\[ = \left( \frac{60840}{76160} \right)^{\frac{1}{4}} = 0.8518 \] m

\[ T_{4000 \text{m}} = \frac{60840}{0.8518 \times 2.86} = 24.9 \text{K} \]

\[ T = \frac{24.9 \text{K}}{0.8518} = -24.2^\circ \text{C} \]

**Alternatively, in a dry adiabatic model,**

\[ T = T_0 - B z \] 

where \( B = 0.0098 \text{ K/m} \) (lapse rate)

\[ T = 15^\circ - 0.0098 \times 4000 = -24.2^\circ \text{C} \]
Problem 1.19

Given: Air in automobile tire modelled as in sketch. Air in tire initially at 25°C and 202 kPa and air added until air at 30°C and 303 kPa.

Find: Mass of air added to tire. Air pressure after air cooled down to 0°C.

Solution: The mass of air added to the tire is the difference of the final mass of air \( M_f \) and the initial mass \( M_i \). Assuming air is an ideal gas,

\[ M_f - M_i = \left( \frac{P_f V}{RT_f} \right) - \left( \frac{P_i V}{RT_i} \right) = \frac{V}{R} \left( \frac{P_f}{T_f} - \frac{P_i}{T_i} \right). \]

Now,

\[ V = \pi \left( r_2^2 - r_1^2 \right) \text{cm}^3 = \pi \left[ (26 \text{ cm})^2 - (16.5 \text{ cm})^2 \right] \left( 13 \text{ cm} \right) \left( \frac{100 \text{ cm}}{1 \text{ m}} \right)^3 = 0.0145 \text{ m}^3. \]

Using Table A.3 \( R = 8.31 \text{ Nm/kgK} \) for air,

\[ M_f - M_i = \left( \frac{0.0145 \text{ m}^3}{287 \text{ Nm/kgK}} \right) \left[ \frac{303 \text{ kPa} - 202 \text{ kPa}}{(273 + 30) \text{ K} - (273 + 25) \text{ K}} \right] \left( \frac{100 \text{ m}^3}{\text{ kPa}} \right). \]

\[ M_f - M_i = 0.0185 \text{ kg}. \]

Now consider the cooling process. The initial state will be 30°C and 303 kPa. The final state will be 0°C and \( P_f \). Applying the ideal gas law to both states gives

\[ \left( \frac{P_f V_f}{RT_f} \right) = \left( \frac{P_i V_i}{RT_i} \right). \]

Since \( V_f = V_i \),

\[ P_f = P_i \left( \frac{T_f}{T_i} \right) = (303 \text{ kPa}) \left( \frac{273 + 0}{273 + 30} \right) = P_f = 273 \text{ kPa}. \]

\[ \text{[II]} \]
**PROBLEM 1.60**

Given water rises to a height $h$ in a vertical tube placed in the water. See Fig. P1.60.

**SOLUTION**

Consider a free body diagram of the liquid surface. Summing forces in the vertical direction gives

$$\sum F = 0$$

or

$$\pi d^2 \cos \theta + (P - P_{atm}) \frac{\pi d^2}{4} = 0.$$

Using the above equation for $p$ gives

$$P - P_{atm} = \gamma h$$

so

$$\pi d^2 \cos \theta = \gamma h \left( \frac{\pi d^2}{4} \right) = 0,$$

$$\cos \theta = \frac{\gamma h d}{4}$$

or

$$\theta = \cos^{-1} \left( \frac{\gamma h d}{4} \right).$$

**Note:**

Alternatively:

Balance of forces on the liquid column gives:

- Weight $W = \gamma h$ vertical component of surface tension force
- $\gamma \left( \frac{\pi d^2}{4} \right)$

or

$$\theta = \cos^{-1} \left( \frac{\gamma h d}{4} \right).$$