1. The manometer reading is 150 mm when the tank is empty (water surface at \( A \)). Calculate the manometer reading when the tank is filled with water.

2. In the dry adiabatic model of the atmosphere, in addition to the state equation for a perfect gas, air is also assumed to obey the equation:

\[
p / \rho^n = \text{constant}
\]

where \( n = 1.4 \). If the conditions at sea level are standard (101.3 kPa, 15°C), determine the pressure and temperature at an altitude of 4000 m above sea level.

3. Calculate the \( h \) at which this gate will open.

4. A solid cylinder 1.0 m in diameter is used as an automated gate valve, as shown in the arrangement below. It is designed to open by pivoting about the hinge \( A \) when the water level \( H \) exceeds 4.5 m. Determine the required mass per unit length of the cylinder.

Answer: 2197 kg/m
Referring to the figure shown.

Let $CE = 150\text{mm}$ be the manometer reading when the tank is empty (water surface at A).

Let $KF$ be the new reading when tank is full.

Then due to conservation of mass: $EF = BG$

or $KF = 0.150\text{m} + 2x$

Now $p_B = p_C$; $p_B = p_A + \gamma \cdot h = 0\text{ (gauge)} + 1000 \times 9.8 \times h$

$= p_C = p_E + \gamma_{Hg} (0.15) = 0\text{ (gauge)} + 1.36 \times 1000 \times 9.8 \times (0.15) = 20\text{ m}$

\[ h = \frac{20000}{1000 \times 9.8} = 2.04\text{ m} \]

New with tank full: $p_A = p_C$ (same level, same fluid)

\[ p_A = p_d + \gamma_{H_2O} (h + 3 + x) \quad ; \quad p_d = 0\text{ (gauge)} \]

\[ p_C = p_F + \gamma_{Hg} (0.15 + 2x) \quad ; \quad p_F = 0\text{ (gauge)} \]

\[ 0 + 1000 \times 9.8 (2.04 + 3 + x) = 0 + 13.6 \times 1000 \times 9.8 (0.15 + 2x) \]

\[ x = 0.118\text{ m} \]

\[ \therefore \text{New reading} = KF = 0.150 + (0.118)^2 = 0.386\text{ m} \]
\[
\frac{dp}{dz} = -pg
\]

\[
dp/p = -gdz
\]

New \[ P/p^n = \text{constant} \ C \Rightarrow p = \left(\frac{P}{C}\right)^{\frac{1}{n}} \]

\[
c = \left(\frac{P}{p^n}\right)_\text{sea level}
\]

At sea level: \[ \frac{P}{p} = RT \Rightarrow p = \frac{P}{RT} \; ; \; R = 286.8 \; \text{Kg/m}^3 \text{ for air} \]

\[
\Rightarrow p = \frac{101300}{(286.8 \times (273+15))}
\]

\[
= 1.226 \; \text{Kg/m}^3
\]

\[
c = \frac{101300}{1.226} = 76660
\]

\[
p = \left(\frac{P}{76660}\right)^{\frac{1}{1.4}} = P/3069
\]

\[
= 0.0003258 \; P^{0.7143}
\]

\[
\frac{dp}{dz} = 9.8 \; \text{d}z
\]

or

\[
\frac{dp}{P^{0.7143}} = \frac{313.2}{d}z
\]

\[
313.2 \left[ \frac{p}{P^{0.7143}} \right]^{3} = -\int_{0}^{\infty} dp/dz \; dz
\]

\[
313.2 \left[ \frac{p}{P^{0.7143}} \right]^{3} = \left[ \frac{p}{P^{0.7143}} \right]_{0}^{\infty}
\]

\[
313.2 \left[ \frac{0.2857}{P^{0.7143}} \right] = -4000 + 0
\]

\[
0.2857 = \left[ -4000 \times 0.2857/313.2 + 101300 \right] = 0.2857
\]

\[
P_{4000} = \left[ -4000 \times 0.2857/313.2 + 101300 \right] = 0.2857
\]

\[
\therefore P_{4000} = 60.24 \; \text{Pa} = 60.24 \; \text{KPa}
\]
But \[ P_{@4000\text{m}} = \left( \frac{P_{@4000\text{m}}}{C} \right)^{\frac{1}{1.4}} \]

\[ = \left( \frac{60840}{76160} \right)^{\frac{1}{1.4}} = 0.8518 \frac{\text{kg}}{\text{m}^3} \]

\[ T_{@4000\text{m}} = \frac{60840}{(0.8518 \times 2.868)} = 24.9^\circ \text{K} \]

\[ T = 24.9^\circ \text{K} = -24^\circ \text{C} \text{ (\star)} \]

\[ T = T_0 - Bz \quad \text{where} \quad B = 0.0098 \frac{\text{K}}{\text{m}} \text{ (lapse rate)} \]

\[ T = 15^\circ \text{C} - 0.0098 \times 4000 = -24.2^\circ \text{C} \]

\[ \text{At} \]

\[ \text{\star} \text{ Alternatively, in a dry adiabatic model,} \]

\[ \text{\star} \text{\star} \text{\star} \text{\star} \text{\star} \text{\star} \]
Let \( l \) be the length (into the page) of the concrete gate.

Then the forces acting on the gate are the horizontal force \( F_H \) acting on the plane area \( AB \) through \( P \), the vertical force \( F_V \) on plane area \( AD \), and the gate's own weight \( W \) acting through its centre of gravity \( G \), as shown.

Now \( F_H = (\rho g l_c)(h \times l) = (\text{density} \cdot g \cdot \frac{l_c}{2})(h \cdot l) \).

Location of \( F_H \): \( CP = \frac{l_c}{2} = \frac{l_a h^3/12}{h} = \frac{l_a h^2}{12} \).

Distance \( OP = OB + BC + CP = (1.2 - l) + \frac{h}{2} + \frac{h}{2} = 1.2 - \frac{h}{3} \) (i.e., \( AP = \frac{h}{3} \)).

\( F_V = (\text{Pressure at } A) \times (\text{Area of } AD) = (\text{water depth})(0.9 \times l) \).

This force acts through centre of pressure \( F_V \) of \( AD \), which coincides with its centroid.

Hence, \( AK = 0.9/2 = 0.45 \text{ m} \).

When the gate starts to open,

Moment (about hinge \( D \)) due to \( W \) = Moments due to \( F_H \) & \( F_V \).
\[ W \times \frac{2}{3} \times (0.9) = F_x \cdot \overline{xy} + F_y \cdot \overline{AC} \]

\[ W = \frac{1}{2} \times (1.2 \times 0.9 \times l) \times 24000 = 12960 \text{ N} \]

\[ 12960 \times \frac{2}{3} \times (0.9) l = \left[ 1000 \cdot 9.8 \times \frac{l^2}{2} \cdot l \right] (1.2 - \frac{h}{3}) \]

\[ + (1000 \cdot 9.8 \times h \times 0.9 \times l) (0.45) \]

Cancelling \( l \) from both sides:

\[ 12960 \times 0.3 = 4900 \cdot h^2 (1.2 - \frac{h}{3}) + 8820 \cdot h (0.45) \]

Solving for \( h \) gives: \( h = 0.57 \text{ m} \) \( (\theta) \)

Note: the other two solutions (\( h = -1.03 \text{ m} \) and \( h = 4.05 \text{ m} \)) are rejected on physical grounds: \( h = -1.03 \text{ m} \) corresponds to the water level completely below the gate; whereas \( h = 4.05 \text{ m} \) requires an extension of the side \( AO \) of the gate - this without changing its weight!