Question 1

In the arrangement shown, a Pitot probe is used for measuring the total pressure and is immersed in a supersonic air stream. It reads 400 kPa (absolute). A shock wave is formed just ahead of the probe's head hole, as shown. A temperature sensor registers 560 K at the probe's head. The static pressure ahead of the shock, measured through a wall tap, reads 80 kPa (absolute). Determine the Mach number, velocity and temperature of the flow. Make reasonable assumptions.

\[
\text{Ans: } M = 1.87, \quad V = 681 \text{ m/s}, \quad T = 330 \text{ K}
\]

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Question 2.

Air flows isentropically through a convergent-divergent nozzle. At a section in the convergent part of the nozzle, the area is 0.1 m², pressure (static) 600 kPa, (absolute) temperature 20°C and the flow Mach number is 0.6. Determine the area, pressure and temperature at a section in the divergent part of the nozzle for a local Mach number of 3.0.

\[
\text{Ans: } A = 0.357 \text{ m}^2, \quad T = 112 \text{ K}, \quad P = 20.8 \text{ kPa (abs.)}
\]
1 = Upstream of shock
2 = Downstream of shock
0 & 3 = Stagnation state (or Total state)

\[
\frac{p_0}{p_1} = \frac{400}{80} = 5
\]

\[
p_0 = \frac{p_2}{p_1} \cdot p_1
\]

\[
\text{Adiabatic flow before shock } \Rightarrow \frac{p_0}{p_1}.
\]

\[
\frac{p_2}{p_1} = \left[ \frac{2.4M_1^2}{2 + 0.6M_1^2} \right]^{9.5} \left[ \frac{2.4}{2.8M_1^2 - 0.4} \right]^{9.5} \frac{p_1}{1 + 0.2M_1^2}
\]

\[
= 5
\]

Solving for \( M_1 \) by graph:

\[
M_1 = 1.87
\]

\[
\alpha_1 = \sqrt{\frac{kR}{T_1}} \text{, but } \frac{T_0}{T_1} = 1 + 0.2M_1^2
\]

\[
\text{Also, across shock } T_0 = T_2 = 560 \text{ K}
\]

\[
T_1 = \frac{T_0}{1 + 0.2M_1^2} = \frac{560}{1 + 0.2(1.87)^2} = 380 \text{ K}.
\]

\[
\alpha_2 = \sqrt{14 \times 287 \times 380} = 964 \text{ m/s} \Rightarrow \nu_1 = M_1 \alpha_1 = 1.87 \times 366
\]

\[
= 681 \text{ m/s}
\]
\[ A_2 = (4.24) \frac{1}{1.19} = 0.357 \text{ m}^2 \quad \circled{8} \]

\[ \frac{A_2}{A_1} = \frac{1}{0.6} \frac{\rho_{o2}}{\rho_{o1}} \]

\[ \frac{\rho_2}{\rho} = \left(\frac{\rho_2}{\rho_{o2}}\right) \left(\frac{\rho_{o1}}{\rho_1}\right) \quad \text{Diatomic} \]

\[ \rho_o = \left(1 + 0.2M^2\right)^{3.5} \Rightarrow \frac{\rho_o}{\rho_1} = 1.276 \]; \[ \frac{\rho_o}{\rho_2} = 36.73 \]

\[ \rho_2 = 600 \times 1.276 / 36.73 = 20.4 \text{ at the Celcius} \quad \circled{9} \]