

**Lecture 2B – Magnetism**


**Magnetic Dipole Moment**

The magnetic field produced by a loop of wire (obtained using the Law of Biot-Savart) looks similar to that of a magnet. Therefore, a current loop can be considered to be like a small permanent magnet, and it will have a magnetic dipole moment (similar to the electric dipole moment).

The torque on a current loop immersed in a magnetic field is given by:

\[ T = I A \times B \]  

To make this similar to the torque experienced by an electric dipole, we define the magnetic dipole moment to be:

\[ m = I A \]  

The torque experienced by a current loop due to an external field can then be expressed as:

\[ T = m \times B \]
This torque will tend to align a magnetic dipole in the direction of an applied field.

An atom with an orbiting electron can be modelled as a current loop.

A permanent magnet is made of many molecular magnetic dipole moments that align in the same direction:

![Diagram of molecular current loops aligning with an equivalent surface current loop](image)

This is why the field of a solenoid looks like that of a magnet.

**Magnetisation**

The tiny magnets created by circulating atomic currents are the sources of the B field of permanent magnets and magnetisable materials. Experiment shows that certain materials (called magnetic materials), when placed in a magnetic field, react upon it and modify it. This phenomenon is called magnetisation.

The magnetization is defined as the average dipole moment per unit volume:

\[
M = \frac{m}{V}
\]  

(2B.4)

Remember the concept of polarization. The magnetization provides a link between the microscopic (m) and the measurable (M).
A magnetic material that is placed in a magnetic field will become magnetized. The material then contributes to the external field. A measure of this induced effect (like polarization) is the magnetization. In ferromagnetic materials, the induced $M$ remains after the external field is withdrawn.

This explains why a rod of steel that is inserted into a solenoid increases the field.

The magnetic field $B$ is modified by the induced $M$:

$$B = \mu_0 (H + M) = \mu H$$

$$\mu = \mu_0 \left(1 + \frac{M}{H}\right) = \mu_0 \mu_r$$

Magnetic materials are classified into three groups:

(i) diamagnetic ($\mu_r \approx 0.999$). eg. molecular hydrogen, water, copper, glass.

(ii) paramagnetic ($\mu_r \approx 1.001$). eg. molecular oxygen, aluminium.

(iii) ferromagnetic ($\mu_r \geq 100$). eg. iron, nickel, cobalt.

![Magnetisation](image)

**Figure 2B.3**
Above a certain temperature $T_C$ (called the Curie temperature), ferromagnetism disappears and ferromagnetic materials become paramagnetic.

There are two possible causes of magnetism:

(i) electron orbital motion around the nucleus.

(ii) electron spin (about own axis).

**Diamagnetism**

Diamagnetism is essentially a quantum mechanical phenomenon. To do a "classical" analysis that agrees with observed results, we have to assume that electrons are paired in orbits and move in opposite directions at the same speed. Without an applied field, there is no net magnetic moment.

Consider one orbiting electron:

![Figure 2B.4](image)

The electron is in equilibrium in its orbit. An electric centripetal force holds the electron to its atom:

$$F_e = m_e a = m_e \omega_0^2 r$$  \hspace{1cm} (2B.6)

Application of a magnetic field $\mathbf{H}$ exerts an additional magnetic force on the electron (a Lorentz force). The radius of the electron orbit does not change, since we are using the Bohr model of the atom. The direction of the Lorentz
force depends on the direction of the magnetic field. Assume an $H$ field direction that slows down the electron:

![Figure 2B.5](image)

Newton's second law gives, for the new angular velocity:

$$F_e - F_m = m_e a$$

$$m_e \omega_0^2 r - e \omega r \mu_0 H = m_e \omega^2 r$$

$$-e \omega \mu_0 H = m_e (\omega - \omega_0)(\omega + \omega_0)$$  \hspace{1cm} (2B.7)

Since the change in speed will be small, then:

$$\omega - \omega_0 = \Delta \omega$$

$$\omega + \omega_0 \approx 2 \omega$$

$$\Delta \omega \approx -\frac{e \mu_0}{2m_e} H$$  \hspace{1cm} (2B.8)

The decrease in electron speed is proportional to the applied field. The electron orbiting in the opposite direction would speed up. The resultant effect is to reduce the field in the material.

Diamagnetism reduces the $B$ field
Paramagnetism

Electrons not only have orbital motion, but spin motion as well.

![Diagram of electron orbit and spin](image)

Each spinning electron produces a spin magnetic moment. Due to thermal vibrations, the axes of the spins are randomly distributed over all possible orientations. A piece of paramagnetic material has no net external magnetization.

An applied $H$ field will tend to align theses magnetic moments in its direction. The alignment is opposed by thermal agitation which for paramagnetic and diamagnetic materials is much stronger. The result is a very slight increase in the magnetic field in the material.

Ferromagnetism

Inner shells (close to the nucleus) of a ferromagnetic atom have unpaired electrons, which are shielded from the influence of other atoms. Each molecule therefore exhibits a strong resultant spin magnetic moment. The strong field of the molecular dipoles causes them to align over small volumes called domains. (A domain has a typical dimension between $10^{-3}$ and $10^{-6}$ m, and contains about $10^{16}$ atoms. They were discovered by Weiss in 1906).

Normally the domains are oriented at random and are not noticeable externally. When an external $H$ field is applied, the dipoles try to align with $H$ and domains with $M$ in the direction of $H$ grow at the expense of the others.
“Saturation” is reached when no further dipole alignment is possible. A strong \( B \) field results. On removal of the applied field, some magnetization is retained (the domains do not return to their original state).

The \( B-H \) Characteristic (Hysteresis)

A piece of ferromagnetic material without any applied fields has the following microscopic structure:

![Figure 2B.7](image)

There are large internal fields that cause the molecular dipoles to align in regions called domains. Adjacent domains are oriented so that the magnetic field lines form closed loops \textit{easily} (using the minimum of energy).
If we apply a large external $\mathbf{H}$ field to the material then three things happen:

(i) magnetic dipoles tend to align with the applied field in directions of “easy” magnetisation (those directions that line up with the crystal structure of the material). Removal of the field causes the dipoles to turn back to their original state – the process is reversible:

(ii) domains in the general direction of the applied field grow at the expense of others. This involves movement of the domain walls – it takes energy and is irreversible. Eventually, there is just one domain:
(iii) the magnetic dipoles align with the applied field (called “hard” magnetisation because generally the field does not line up with the crystal structure of the material) until all dipoles are aligned – saturation has been achieved.

Finally, the dipoles turn in the direction of “hard” magnetisation.
To observe the way a $B-H$ characteristic is traced out, we can use a toroidal specimen (*Why a toroid?*) and direct current:

![Diagram of applied H and ferromagnetic material](image)

**Figure 2B.12**

The steps to obtain the $B-H$ characteristic are:

(i) $H$ (or $I_{dc}$) is gradually increased. $B$ in the material increases along $oa$ until no further alignment is possible (saturation is reached at $H_{\text{max}}$).

(ii) $H$ (or $I_{dc}$) is reduced. $B$ decreases along $ab$ (not $ao$). This property is known as hysteresis (Greek: short coming). No part of the magnetization curve is now reversible.

(iii) $H$ is further reduced to $H_{\text{min}}$ and then increased again. $B$ follows the path $bcdefa$. (N.B. the path terminates at $a$ only if we apply $H_{\text{max}}$ again).

**Minor Loops**

If we are at point $c'$ (say) and $H$ is increased (made more positive) and then decreased to the previous value, the minor loop shown in Figure 2B.11 is traced.
The Normal Magnetization Characteristic

Different $B$-$H$ loops are obtained for different values of $H_{\text{max}}$. Joining the tips of each hysteresis loop gives the “normal magnetization characteristic”:

![Figure 2B.13](image)

The normal magnetization characteristic is used often. It is like an average characteristic, but it doesn't tell us about the shape of the hysteresis loop (and therefore the losses). It is well suited to analysis where AC excitation of the material is involved. (Why?)
Summary

- A magnetic dipole is a current loop. A magnetic dipole experiences a torque when subjected to an external magnetic field.

- An atom with an orbiting electron can be modelled as a current loop, i.e. as a magnetic dipole.

- A magnetic dipole placed in a magnetic field experiences magnetisation, which acts to increase the relative permeability – for an inductor, the inductance will increase due to the magnetisation.

- Magnetic materials can be categorised into three groups: diamagnetic, paramagnetic and ferromagnetic.

- Diamagnetism is caused by orbiting electrons – it reduces the $B$ field slightly.

- Paramagnetism is caused by spinning electrons – it increases the $B$ field slightly.

- Ferromagnetism is cause by unpaired spinning electrons – it increases the $B$ field significantly.

- A ferromagnetic material’s magnetic properties can be described with a $B$-$H$ characteristic, which exhibits hysteresis. The $B$-$H$ characteristic is a result of the physical crystal structure which is divided into domains.

References

