Weir Slurry Pumping Manual

WEIR SLURRY PUMPING MANUAL

Foreword

Transportation of bulk materials in the form of a slurry through piping systems is a most efficient and effective mechanism. Piping systems can take many forms from overland pipelines to mineral processing plants, coal washeries and the like.

This Manual sets out the basic principles and gives a good appreciation of the technology. It should be useful to both experienced practitioners and relative newcomers by providing a comprehensive understanding of the principles necessary for developing, maintaining or appraising slurry systems or proposals. It is written in plain English with minimal use of technical jargon whilst retaining the terminology necessary to provide a good scientific engineering basis.

I have no hesitation in recommending and supporting the use of the Manual as a helpful tool for practitioners in the bulk materials handling industry.

[Signature]

Professor Allan C. Wightley
General Manager
Weir Slurry Group Technology

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Acknowledgements

The authors of this Manual are:

Anthony (Tony) Grzina – formerly Engineering Services Manager
Warman International Ltd (now a Consultant)

Aleks Roudnev – Principal Specialist – Applied Hydraulics
Weir Slurry Group, Inc.

Kevin E. Burgess – Manager – Product Design
Warman International Ltd.

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Symbols used

The terms slurry and mixture in this Manual are used interchangeably to describe a mix of any loose solids, made up in any proportions and combinations of any particle sizes and any conveying liquid. The subscript w refers to densities and specific gravities of liquids – mostly, but not exclusively, water. The letter w has been used in preference to the letter l, because l can be confused with the digit l. The term water is often used here when the term liquid would be more correct. On the rare occasion when it is essential to differentiate between water and some other liquid, then both w and l should be used as needed. Note that w is also used to designate a function of weight or mass, as in concentration by weight.

The subscript m identifies the density and specific gravity of a slurry. Often, to avoid repetition, the words head of fluid are used in units of head measurements when the word fluid refers to either liquid (when pumping a liquid) or slurry (when pumping a slurry). The subscripts s and d are used to identify pipes, valves, gauges or physical conditions respectively at, or upstream of, pump suction flange and at, or downstream of, discharge flange. The equal sign (=) is used to indicate that the character in the Symbols column is actually a dimension.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Pipe wall roughness factor in Hazen-Williams pipe friction equation</td>
<td>dimensionless</td>
</tr>
<tr>
<td>Cc</td>
<td>Concentration of coarse solid particles in slurry, by weight – smallest size of solids must be defined</td>
<td>%</td>
</tr>
<tr>
<td>Cf</td>
<td>Concentration of fine solid particles in slurry, by weight – largest size of solids must be defined</td>
<td>%</td>
</tr>
<tr>
<td>Cv</td>
<td>Concentration of solids in slurry, by true volume</td>
<td>%</td>
</tr>
<tr>
<td>Cw</td>
<td>Concentration of solids in slurry, by weight</td>
<td>%</td>
</tr>
<tr>
<td>D</td>
<td>Inside diameter of pipe</td>
<td>m</td>
</tr>
<tr>
<td>Dd</td>
<td>Inside diameter of pump discharge pipe</td>
<td>m</td>
</tr>
<tr>
<td>Di</td>
<td>Impeller diameter. Sometimes expressed in mm</td>
<td>m</td>
</tr>
<tr>
<td>Dj</td>
<td>Inside diameter of nozzle used in jet-stacking of solids</td>
<td>m</td>
</tr>
<tr>
<td>dmax</td>
<td>Maximum solid particle size in a slurry</td>
<td>m (also mm, µm)</td>
</tr>
<tr>
<td>Ds</td>
<td>Inside diameter of pump suction pipe</td>
<td>m</td>
</tr>
<tr>
<td>dv/dy</td>
<td>Velocity gradient between fluid layers in pipe under laminar flow. Also known as shear rate</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>d₅₀</td>
<td>Median diameter of a sample of dry solid particles. It is the particle size equal to the screen aperture, which would pass exactly 50% by weight of the sample. Note that d₅₀ is not the mean, or average, particle size of the sample</td>
<td>m (also mm, µm)</td>
</tr>
<tr>
<td>d₂₀</td>
<td>For the same sample as above, it is the particle size equal to the screen aperture which would pass exactly 20% by weight of the sample</td>
<td>m (as above)</td>
</tr>
<tr>
<td>d₈₀</td>
<td>For the same sample as above, it is the particle size equal to the screen aperture which would pass exactly 80% by weight of the sample</td>
<td>m (as above)</td>
</tr>
<tr>
<td>d₅₀/d₂₀</td>
<td>Fineness modulus (FM) – Term borrowed from concrete mixing industry. It indicates particle size distribution in a sample, e.g. sample with FM=5 has wide variety of sizes; sample with FM=2 has a narrow variety of sizes</td>
<td>dimensionless</td>
</tr>
<tr>
<td>em</td>
<td>Efficiency of pump when handling slurry</td>
<td>%</td>
</tr>
<tr>
<td>ER</td>
<td>Efficiency ratio of a pump: em/ew at constant flow rate and pump speed</td>
<td>dimensionless</td>
</tr>
<tr>
<td>ew</td>
<td>Efficiency of pump when handling water</td>
<td>%</td>
</tr>
<tr>
<td>f</td>
<td>Darcy’s pipe friction factor</td>
<td>dimensionless</td>
</tr>
<tr>
<td>Fₒ</td>
<td>Durand’s parameter for limiting settling velocity in a pipe</td>
<td>dimensionless</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational acceleration</td>
<td>9.81 m/s²</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>h</td>
<td>General symbol for head used for sundry purposes, defined and with dimensions specified as needed</td>
<td>m fluid</td>
</tr>
<tr>
<td>H</td>
<td>Total head required by a pumping system also: Total head developed by a pump</td>
<td>m fluid</td>
</tr>
<tr>
<td>Hatm</td>
<td>Local atmospheric pressure expressed as a head</td>
<td>m fluid</td>
</tr>
<tr>
<td>Hc</td>
<td>Head, equivalent of pressure required to operate a hydro-cyclone, a flow distribution box or other pressurised vessel</td>
<td>m slurry</td>
</tr>
<tr>
<td>Hd</td>
<td>Total pump discharge head, referred to pump centreline – usually positive</td>
<td>m fluid</td>
</tr>
<tr>
<td>Hf</td>
<td>Friction head loss in pipe</td>
<td>m fluid</td>
</tr>
<tr>
<td>Hf₀₀</td>
<td>Relative friction head loss in pipe</td>
<td>m fluid/100m pipe</td>
</tr>
<tr>
<td>Hfd</td>
<td>Friction head loss in discharge pipe</td>
<td>m fluid</td>
</tr>
<tr>
<td>Hfs</td>
<td>Friction head loss in suction pipe</td>
<td>m fluid</td>
</tr>
<tr>
<td>Hgd</td>
<td>Head at pump discharge, referred to pump centreline – positive if above atmospheric head and negative if below</td>
<td>m fluid</td>
</tr>
<tr>
<td>Hgs</td>
<td>Head at pump suction, referred to pump centreline – positive if above atmospheric head and negative if below</td>
<td>m fluid</td>
</tr>
<tr>
<td>Hi</td>
<td>Head loss at inlet to suction pipework also: Combined head loss in suction pipework</td>
<td>m fluid</td>
</tr>
<tr>
<td>Hm</td>
<td>Total head required by a slurry pumping system also: Total head developed by a pump when handling slurry</td>
<td>m slurry</td>
</tr>
<tr>
<td>HR</td>
<td>Head ratio of a pump: Hm/Hw at constant flow rate and pump speed</td>
<td>dimensionless</td>
</tr>
<tr>
<td>Hpr</td>
<td>Gauge head equivalent of pressure [kPa] (above atmospheric pressure) inside a closed pump supply vessel</td>
<td>m fluid</td>
</tr>
<tr>
<td>Hs</td>
<td>Total pump suction head, referred to pump centreline – positive or negative</td>
<td>m fluid</td>
</tr>
<tr>
<td>Hvac</td>
<td>Gauge head equivalent of vacuum [kPa] (below atmospheric pressure) inside a closed pump supply vessel</td>
<td>m fluid</td>
</tr>
<tr>
<td>Hvap</td>
<td>Absolute head equivalent of vapour pressure [kPa] of conveying liquid at pumping temperature</td>
<td>m fluid</td>
</tr>
<tr>
<td>Hv</td>
<td>Velocity head of flowing fluid in pipe (=V²/2g)</td>
<td>m</td>
</tr>
<tr>
<td>Hw</td>
<td>Total head required by a water pumping system also: Total head developed by a pump when handling water</td>
<td>m water</td>
</tr>
<tr>
<td>kPa</td>
<td>= kilo Pascal = 1000 Pascal = 1000 Newton/m², multiple unit of pressure</td>
<td>–</td>
</tr>
<tr>
<td>L</td>
<td>Total equivalent length of pipe, including actual length of pipe and equivalent lengths for valves, bends and fittings</td>
<td>m</td>
</tr>
<tr>
<td>L =</td>
<td>Litre, unit of volume</td>
<td>0.001m³</td>
</tr>
<tr>
<td>Lᵃ</td>
<td>Actual length of pipe</td>
<td>m</td>
</tr>
<tr>
<td>Lᵇ</td>
<td>Equivalent length of pipe for a bend</td>
<td>m</td>
</tr>
<tr>
<td>Lᶠ</td>
<td>Equivalent length of pipe for a pipe fitting</td>
<td>m</td>
</tr>
<tr>
<td>Lʲ</td>
<td>Horizontal distance reached by slurry jetted from a nozzle inclined at an angle θj above horizontal</td>
<td>m</td>
</tr>
<tr>
<td>Lᵛ</td>
<td>Equivalent length of pipe for a valve</td>
<td>m</td>
</tr>
<tr>
<td>M</td>
<td>Mass flow rate of dry solids in slurry</td>
<td>t/h</td>
</tr>
<tr>
<td>M</td>
<td>Mass of a Sample of Solids</td>
<td>kg</td>
</tr>
<tr>
<td>Mᵐ</td>
<td>Mass flow rate of slurry</td>
<td>t/h</td>
</tr>
<tr>
<td>Mʷ</td>
<td>Mass flow rate of liquid in slurry</td>
<td>t/h</td>
</tr>
<tr>
<td>N</td>
<td>Pump rotational speed</td>
<td>r/min</td>
</tr>
<tr>
<td>N =</td>
<td>Newton, unit of force</td>
<td>kg.m/s²</td>
</tr>
<tr>
<td>NPSH</td>
<td>Net positive suction head, at pump suction flange, at given flow rate and temperature</td>
<td>m fluid</td>
</tr>
<tr>
<td>NPSHa</td>
<td>NPSH available at pump suction flange, at given flow rate and temperature</td>
<td>m fluid</td>
</tr>
<tr>
<td>NPSHr</td>
<td>NPSH required at pump suction flange, at given pump speed, flow rate and temperature to prevent cavitation</td>
<td>m fluid</td>
</tr>
<tr>
<td>Nᵣ</td>
<td>Reynolds number for flow in pipe = ρDV/µ (the symbol Re is also used)</td>
<td>dimensionless</td>
</tr>
<tr>
<td>P</td>
<td>Pressure, general symbol</td>
<td>Pa (also kPa, Mpa)</td>
</tr>
<tr>
<td>Pa</td>
<td>= Pascal, unit of pressure</td>
<td>N/m²</td>
</tr>
<tr>
<td>Patm</td>
<td>Atmospheric pressure – a function of elevation above sea level</td>
<td>kPa</td>
</tr>
<tr>
<td>Pᵣ</td>
<td>Power input to pump shaft</td>
<td>kW</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit/Notation</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>---------------</td>
</tr>
<tr>
<td>$P_{vap}$</td>
<td>Vapour pressure of liquid – a function of its temperature</td>
<td>kPa</td>
</tr>
<tr>
<td>$Q$</td>
<td>Flow rate of fluid in pipe</td>
<td>L/s or m³/h</td>
</tr>
<tr>
<td>$r/min$</td>
<td>Rotational speed of pump = rotations per minute = rpm</td>
<td>–</td>
</tr>
<tr>
<td>$Re$</td>
<td>Specific gravity (SG) of dry solids = $p/p_w$</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$Sw$</td>
<td>SG of liquid, usually water at 20°C = 1</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$Sm$</td>
<td>SG of slurry = $p_m/p_w$</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$T$</td>
<td>Water temperature</td>
<td>°C</td>
</tr>
<tr>
<td>$t$</td>
<td>$1000 \times$ unit of mass (1kg)</td>
<td>tonne</td>
</tr>
<tr>
<td>$t/h$</td>
<td>Mass flow rate of solids, liquid or slurry</td>
<td>tonnes/hour</td>
</tr>
<tr>
<td>$v$</td>
<td>Local velocity of thin fluid shell in pipe under laminar flow</td>
<td>m/s</td>
</tr>
<tr>
<td>$v_c$</td>
<td>Maximum local velocity in laminar flow (along pipe centreline)</td>
<td>m/s</td>
</tr>
<tr>
<td>$V$</td>
<td>Average velocity of flow in pipe</td>
<td>m/s</td>
</tr>
<tr>
<td>$V_c$</td>
<td>Wilson’s maximum deposition velocity for solids in pipe</td>
<td>m/s</td>
</tr>
<tr>
<td>$V_d$</td>
<td>Critical average velocity of flow in pipe, at $N_R=2000$</td>
<td>m/s</td>
</tr>
<tr>
<td>$V_{ic}$</td>
<td>Durand’s limiting solids settling velocity in pipe</td>
<td>m/s</td>
</tr>
<tr>
<td>$V_{OL}$</td>
<td>General symbol for volume</td>
<td>m³ or L</td>
</tr>
<tr>
<td>$V_j$</td>
<td>Exit velocity of slurry from a nozzle</td>
<td>m/s</td>
</tr>
<tr>
<td>$V_s$</td>
<td>Average velocity of flow in suction pipe</td>
<td>m/s</td>
</tr>
<tr>
<td>$Z$</td>
<td>Total static head = vertical distance between liquid level in pump supply vessel to point of delivery level. The latter can be the end of the pipe, if freely discharging in air, or the liquid surface in the receiving vessel, if discharge is submerged. Z is positive or negative depending on whether delivery level is above or below liquid level in supply vessel</td>
<td>m</td>
</tr>
<tr>
<td>$Z_{as}$</td>
<td>Elevation of pump above sea level</td>
<td>m</td>
</tr>
<tr>
<td>$Z_{bc}$</td>
<td>Column of slurry (of $S_m$) in a suction pipe balanced by column $Z_w$ of surrounding liquid (of $S_w$)</td>
<td>m</td>
</tr>
<tr>
<td>$Z_d$</td>
<td>Static discharge head = vertical distance between pump centreline and point of delivery level. $Z_d$ is positive or negative depending on whether delivery level is above or below pump centreline</td>
<td>m</td>
</tr>
<tr>
<td>$Z_j$</td>
<td>Maximum height reached by slurry jetted from a nozzle inclined at an angle $\theta_j$ above horizontal</td>
<td>m</td>
</tr>
<tr>
<td>$Z_s$</td>
<td>Static suction head: vertical distance between liquid level in supply vessel and pump centreline. $Z_s$ is positive or negative depending on whether supply liquid level is above or below pump centreline</td>
<td>m</td>
</tr>
<tr>
<td>$Z_{uc}$</td>
<td>Column of slurry (of $S_m$) in a suction pipe not balanced by surrounding liquid ($=Z_w-Z_{bc}$)</td>
<td>m</td>
</tr>
<tr>
<td>$Z_w$</td>
<td>Vertical projection of a suction pipe full of slurry (of $S_m$), surrounded by liquid (of $S_w$)</td>
<td>m</td>
</tr>
<tr>
<td>$\eta$</td>
<td>(=$\eta$) Coefficient of rigidity of Bingham fluid</td>
<td>Pa.s</td>
</tr>
<tr>
<td>$\eta_m$</td>
<td>(=$\eta_m$) Efficiency of pump when pumping slurry</td>
<td>%</td>
</tr>
<tr>
<td>$\eta_w$</td>
<td>(=$\eta_w$) Efficiency of pump when pumping water</td>
<td>%</td>
</tr>
<tr>
<td>$\mu$</td>
<td>(=$\mu$) Dynamic viscosity of a Newtonian liquid</td>
<td>Pa.s</td>
</tr>
<tr>
<td>$\mu_a$</td>
<td>(=$\mu_a$) Apparent viscosity of a Bingham fluid at given shear rate $dv/dy$</td>
<td>Pa.s</td>
</tr>
<tr>
<td>$\nu$</td>
<td>(=$\nu$) Kinematic viscosity of a Newtonian fluid = $\mu/\rho$</td>
<td>m²/s</td>
</tr>
<tr>
<td>$\rho$</td>
<td>(=$\rho$) Density of solids.</td>
<td>kg/m³ (also t/m³, kg/L)</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>(=$\rho_m$) Density of solids-liquid mixture or of equivalent fluid</td>
<td>kg/m³ (as above)</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>(=$\rho_w$) Density of liquid, usually water</td>
<td>kg/m³ (as above)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>(=$\tau$) Shear stress</td>
<td>Pa</td>
</tr>
<tr>
<td>$\tau_n$</td>
<td>(=$\tau_n$) Yield shear stress of Bingham fluid</td>
<td>Pa</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>(=$\tau_w$) Shear stress at pipe wall</td>
<td>Pa</td>
</tr>
<tr>
<td>$\theta_j$</td>
<td>(=$\theta_j$) Angle of inclined jetting nozzle above horizontal</td>
<td>°</td>
</tr>
</tbody>
</table>
SECTION 2 – Water and Slurry Basics

All matter in the universe exists in one or more of three states: solid, liquid or gaseous and each state is determined by its body temperature and pressure. The best known example of a substance with the three states is water, which in human-survivable temperatures and pressures can exist as ice, water and vapour.

FLUIDS
Liquids and gases are called fluids because they can flow and can take the shape of any container into which they are poured. There are however also considerable differences between liquids and gases. A liquid can have separating upper and lower surfaces with other liquids – if they do not mix – or with gases but a gas cannot have a separating surface with another gas. Some liquids mix with other liquids (water and wine) some do not (water and oil). All gases mix with each other and some gases can dissolve in some liquids. Liquids are virtually incompressible whereas gases are compressible.

Many physical, chemical or other properties identify all substances. In pumping we are mainly concerned with properties associated with mass. A force imparts acceleration to a mass. Weight, as a particular form of force, imparts gravitational acceleration to a mass. For easy comparison of various materials, we usually express their masses relative to a unit volume. This physical property is called Density $\rho$ [kg/m$^3$]. Occasionally density is expressed in tonnes per cubic metre [t/m$^3$] or kg per litre [kg/L], which are numerically equal.

LIQUIDS
The most common liquid handled by centrifugal pumps is water. At normal ambient pressure and at freezing point (0ºC), water and ice have densities of 999 and 895 kg/m$^3$ respectively, which explains why ice floats on water. At the same pressure but at boiling point (100ºC), water and saturated vapour have densities of 957 and 0.590 kg/m$^3$ respectively, which shows that water expands approximately 1600 times after boiling. The most common solid material handled by centrifugal pumps is silica in the form of sand or rock, whose density is around 2650 kg/m$^3$.

Very often we express unit mass of a material by its Specific gravity or SG, a dimensionless number, which we obtain by dividing the density of the material by the density of water. For this particular purpose, we usually take the density of water as 1000 kg/m$^3$ and so the SG of any material is simply equal to its density divided by 1000. By this reasoning, the SG of water is 1 and that of silica is 2.65.

SLURRIES
In solids handling by pumping, we mix some solid matter with some liquid carrier to form a slurry, i.e. a mixture of a liquid and solid particles, large or small. In calculations associated with slurry pumping in this Manual we use the following symbols: the SG of the solids is S, the SG of the liquid is $S_w$. (Note that Sl is preferred by some engineers but the letter “l” can be confused with the digit “1” in some printing fonts). The SG of the mixture is $S_m$. Often mixtures of solids and liquids can be treated as an equivalent fluid with the same SG=$S_m$. Two additional parameters are used, namely Concentration of solids in the mixture by weight $C_w$ [%] and Concentration of solids in the mixture by true volume $C_v$ [%]. The word true is often omitted. To understand its meaning, consider the following:

A hollow cube with sides D has a volume of $D^3$. A solid sphere of diameter D has a volume of $0.52D^3$ and would take up 52% of the cube’s volume. If eight smaller spheres were added with centres at the eight corners of the cube and touching the large sphere the volume of the portions of these spheres inside the cube would be $0.20D^3$ and the combined volume of the nine spheres would be 72% of the cube. A progressive similar fitment of more smaller spheres of correct number and size would make their combined volumes converge quickly to an asymptotic value of around 74%. If we had spheres of the same diameter or if we had solid particles of various shapes and sizes, the true volumes filled would vary from 50% to 80%. For loose beach sand it is around 73%.

There are, as we have seen, five inter-related variables, which we use with slurries. Table 2.1 on next page lists all the equations for calculating any one of the variables from other known values. For water use $S_w=1$. 

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If Cw and Cv values are expressed as percentages (%), divide them by 100 before using them in the tabulated equations.

Table 2.1 – Specific gravity and Concentration equations for Slurries

<table>
<thead>
<tr>
<th></th>
<th>[Sw = \frac{S(SmCw–Sm)}{(SmCw–S)}]</th>
<th>[S = \frac{(SCv–Sm)}{(Cv–1)}]</th>
<th>[S = \frac{S[Cv(Cw–1)]}{[Cw(Cv–1)]}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cw</td>
<td>[= \frac{S(Sm–Sw)}{(Sm–S)}]</td>
<td>[= \frac{Sw+(Sm–Sw)}{Cv}]</td>
<td>[= \frac{SwCw}{(Cw–1+Sw/Sm)}]</td>
</tr>
<tr>
<td>Sm</td>
<td>[= \frac{Sw}{[1–Cw(1–Sw/S)]}]</td>
<td>[= \frac{Sw+Cv(S–Sw)}{Sm}]</td>
<td>[= \frac{Sw(Cv–1)}{(Cw–1)}]</td>
</tr>
<tr>
<td>Cv</td>
<td>[= \frac{(Sm–Sw)}{(S–Sw)}]</td>
<td>[= \frac{Scv}{[Sw+Cv(S–Sw)]}]</td>
<td>[= \frac{1+Sw(Cv–1)}{Sm}]</td>
</tr>
</tbody>
</table>

For any slurries, when any three of the five variables tabulated at left are known, the fourth and fifth may be calculated from the equations shown. Alternatively the Diagram A1-3 in Appendix A1 may be used for the same purpose although that Diagram cannot be used to calculate Sw.

Another useful relationship is:

\[\frac{Cw}{Cv} = \frac{S}{Sm}\] (2.1)

As can be readily seen, this equation is independent of liquid density Sw and it hold true for solids mixed with any liquid. In other words, when the ratio Cw/Cv for a slurry has been given a value, the ratio S/Sm must have the same value and the specific gravity of the liquid Sw can then be changed at will without affecting the said value.

The equation is used to calculate any one of its four variables when the other three are known.
SECTION 3 – Energy and Heads of flowing Liquids

Water and other liquids in a steady state find their own levels. Water, in a dam on a high mountain has potential energy or, said in another way, it has the potential for doing work. On the way down to lower grounds the potential energy converts to kinetic energy, which does the actual work, like driving a turbine-generator set or a water mill or float a barge down a river. Alternatively all this energy can be wasted, like in a waterfall.

Water is stored in dams mostly for the following reasons: (1) in hydroelectric schemes, for electric power generation, (2) in irrigation schemes, for crop cultivation, (3) in town water circulation systems, as one of the essential ingredients of life and (4) in river flood mitigation schemes. Only in hydroelectric schemes must water be stored at high altitudes, where it has lots of convertible potential energy. In irrigation and water circulation schemes, pumps are used to impart kinetic energy to the water to circulate it through the pipes and/or to elevate it to higher grounds.

Fluid flow is always from the point of highest energy to the lowest. Specific Energy is the capacity to do work per unit weight \([\text{N.m/N}]\), which reduces to head in metres \([\text{m}]\). It is important to note and remember that specific energy of a fluid at a given point is numerically equal to its elevation or Potential Head \(Z\) at that point.

If an object falls from a building of height \(H\), it will reach a velocity

\[
V = \sqrt{2gH} \tag{3.1}
\]

just before it hits the ground. Conversely, if the same object is propelled upwards, with an initial velocity \(V\), it will reach a height

\[
H_v = \frac{V^2}{2g} \tag{3.2}
\]

which is another form of energy and it is called Velocity Head.

When dealing with liquids and heads we must invariably deal also with pressures. If we have a vertical cylinder of height \(H\) and diameter \(D\) filled with some liquid, the weight of the liquid is \(W = \rho \cdot g \cdot H \cdot D \cdot \frac{\pi}{4}\). Dividing this by the cross-sectional area of the cylinder \(D^2 \cdot \frac{\pi}{4}\) we get the Static Pressure acting on the bottom of the cylinder:

\[
P = \rho \cdot g \cdot H \tag{3.3}
\]

As we can see, this pressure is independent of the cylinder’s diameter, its shape or volume. Pressure is a function purely of the height and density of the liquid. So, if the liquid is water, a pressure gauge at the bottom of a 10 metres high cylinder shows a pressure \(P=1000 \times 10 \times 9.81 \times 1=98,100\) Pa or \(98.1\) kPa i.e. close to normal atmospheric pressure. If the liquid is mercury (SG= 13.6), the pressure is 13.6 times greater or \(1334\) kPa. In either case, if we move the gauge up the cylinder, the pressure decreases linearly to 0 at the top. From the pressure equation (3.3) we can extract the corresponding Pressure Head:

\[
Hp = \frac{P}{(\rho \cdot g)} \tag{3.4}
\]

A liquid passing through a pipeline at a fixed flow rate follows all directional changes and its local velocities grow and fall inversely with the respective pipe cross-sections. Total head \(H\) (or energy) of the fluid along the pipeline, referred to a horizontal datum plane, is made up of:

(1) the Static head \(Z\), equal to the vertical position of the pipe section,
(2) the Pressure head \(Hp\), obtained from a pressure gauge \(P\) in the pipe or from a liquid column and
(3) the Velocity head \(H_v\), which can be either calculated or obtained by means of a Pitot-Static tube set.
If we neglect internal friction, the sum:

\[ H = Z + H_p + H_v \]

remains constant along the pipeline. This summation is shown in Fig.3.1 and is known as the Bernoulli equation.

It is important to note that:

1. the sums of Z and Hp along the pipeline define the Hydraulic Grade Line (HGL) and
2. the sums of Z, Hp and Hv along the pipeline define the Energy Line (EL).

If we considered internal friction head losses \( H_f \) as well, then the EL would show a continuous drop from left to right in Fig.3.1.

If the pipeline had a constant cross-section, the amount of drop per unit length of pipe would be constant i.e. the EL would be a straight line with a constant falling slope.

On the other hand, if the pipeline had variable cross-sections along the way as shown in Fig.3.1, then the EL would be a curved line with a falling but continuously changing slope.
SECTION 4 – Viscosity and Newtonian Fluids

Any movement of a solid body in our physical world is resisted by external and/or internal friction. Fluid motion is resisted both by internal molecular friction and external, boundary friction. When a shear force is applied at a boundary of a fluid, the latter begins to move in the direction of the force, developing shear stress between adjoining layers. This property of the liquid is called **Viscosity**. If the **Velocity gradient** $dv/dy$ (also known as **Shear rate**) between any adjacent fluid layers is constant, the fluid is called **Newtonian**. The constant of proportionality between the shear stress $\tau$ and the velocity gradient $dv/dy$ is called the **Dynamic Viscosity** $\mu$. The mathematical relationship known as Newton’s law of viscosity is:

$$\tau = \mu \frac{dv}{dy} \quad (4.1)$$

Viscosity changes with temperature and this is shown in Fig.4.1 where $\mu_1$ and $\mu_2$ represent viscosities of the same fluid at different temperatures or viscosities of two different fluids at the same temperature. At higher temperatures, viscosity decreases and the fluid runs “easier”, i.e. with less friction. The best known Newtonian fluids are water and most oils.

When a pressure differential $P$ is applied to a fluid over a pipe length $L$ and diameter $D$, a frictional shear stress $\tau_w$ is generated between the fluid and the pipe at the wall of the pipe. The axial force causing fluid motion is given by $PD^2\pi/4$ and the force resisting this motion, i.e. the wall friction, is given by $\tau_w D\pi L$. By equating these forces and rearranging terms we get the shear stress at the wall:

$$\tau_w = \frac{PD}{4L} \quad (4.2)$$

All of this happens only in laminar flow where adjacent concentric shells (or layers) of fluid slide over each other like the tubes in a telescope and there is virtually no molecular migration across shell boundaries. In turbulent flow on the other hand, while the main flow is axial, there are no actual shell boundaries and molecules roam randomly in all directions. Shear stress $\tau$ reduces linearly from $\tau_w$ at the pipe wall to zero at the centreline under both laminar and turbulent flows.

The solution of the differential equation (4.1) for laminar flow yields a parabolic velocity distribution with local velocities $v=0$ at the wall and $v=v_c$ (maximum) on the centreline, as shown in Fig.4.2. The average velocity of flow $V$ is equal to one half of $v_c$.

Pipe friction loss is also given by the Poiseuille equation $P=32\mu LV/D^2$. Combining this with (4.2) we get $\tau_w=\mu 8V/D$ and by comparing the latter with (4.1) we see that the velocity gradient is equal to:
\[
\frac{dv}{dy} = \frac{8V}{D} \quad (4.3)
\]

Identities 4.2 and 4.3 are very important. We used them as vertical and horizontal scales respectively in Fig.4.1 to determine fluid viscosity from tests. We vary the flow and measure velocities and corresponding pressure losses in a pipe and plot these test points \( \tau_w = PD/4L \) versus \( 8V/D \). We then draw a straight line through all the points and extend it to see if it passes through the origin of the graph. If it does, the fluid is Newtonian. We then obtain the viscosity by calculating the slope of the line. This is done by taking any point, call it ‘z’, on the line – the further from the origin the more accurate the result – and calculate \( \mu = \frac{(\tau_w)z}{(8V/D)z} \).

A dimensionless “Reynolds Number” \( N_R \) (the symbol Re is also used) for flow in pipes, is defined as:

\[
N_R = \frac{\rho DV}{\mu} \quad (4.4)
\]

This number is used mostly in pipe friction factor charts like the Darcy diagram. Laminar flow exists at Reynolds numbers up to 2000. At values between 2000 and 3000, flow is transitional with both forms of flow existing in the pipeline. Pumping at transitional velocities is often avoided to prevent oscillations between laminar and turbulent flows, but for certain types of slurries it is an economical way of transportation. At values above 3000, flow is usually fully turbulent.

A few general observations can be made as follows:
(1) Liquids and liquid-soluble solids are pumped in turbulent flow at low velocities from 0.5 to 2 m/s because they yield the best compromise between pipe friction losses (running costs) and capital costs (pumps and pipework cost)
(2) Slurries of liquids and settling solids are pumped in turbulent flow at higher, safe velocities from 2 to 5 m/s or more, in order to prevent pipe blockages
(3) Slurries of liquids and fine, non-settling solids usually form homogeneous slurries (see Section 5) and are pumped at intermediate velocities between 1 and 3 m/s at Reynolds numbers close to 2000.

**EXAMPLE: ES4.1**

**Reynolds Numbers**

Water at ambient temperature is being pumped through a steel pipe of diameter \( D=0.200 \) m at a flow rate \( Q=30 \) L/s. Water density is \( \rho=998 \) kg/m\(^3\) and dynamic viscosity is \( \mu=10^{-3} \) Pa.s.

Calculate (1) the Reynolds number and (2) find the flow rate in the same pipe at \( N_R=2000 \), i.e. the high end of the laminar flow range.

(1) \( Q=0.030 \) m\(^3\)/s. \[ V=\frac{4Q}{\pi D^2} = \frac{4 \times 0.03}{\pi \times 0.2^2} = 0.95 \text{ m/s} \]

\[ N_R = \frac{\rho D V}{\mu} = 998 \times 0.95 \times 0.2 / 10^{-3} = 1.8 \times 10^5 \]

As we see, a flow velocity of less than 1 m/s yields a Reynolds number much greater than 3000 and well into the turbulent flow range.

(2) \( N_R=2000 \) \[ \frac{V}{\mu N_R (\rho D)} = 10^{-3} \times 2000 / (998 \times 0.2) = 0.01 \text{ m/s} \]

\[ Q=\pi D^2 V / 4 = 0.0003 \text{ m}^3/\text{s} = 0.3 \text{ L/s} \]

This shows what an infinitesimal flow rate we would get if we pumped water even at the highest velocity of the laminar flow region.
SECTION 5 – Bingham Fluids

When high percentages of fine-sized solid particles (of less than 100 µm) are mixed with water, they usually form slurries, which do not behave like Newtonian fluids and in which the solids usually do not settle. There exist various types of non-Newtonian fluids but here we deal only with a group known as Bingham fluids.

A Bingham fluid could be described as a Newtonian fluid with an additional parameter, namely a Yield Stress \( \tau_0 \). This material behaves like a jelly when stationary and like a fluid when moving. If a shear stress below \( \tau_0 \) is applied to it, it flexes like a jelly and when the stress is removed it returns to its original shape. However, if the applied shear stress is above \( \tau_0 \), the material begins to flow. A plot of \( \tau \) against \( dv/dy \) yields a straight line, which intercepts the \( \tau \)-axis at \( \tau_0 \) (greater than zero), and has a slope \( \eta \) known as the Coefficient of rigidity, as shown in Fig.5.1. The mathematical equation of this line is:

\[
\tau = \tau_0 + \eta \cdot dv/dy \quad (5.1)
\]

Arguably the best known examples of Bingham fluids are ketchup and tomato sauce. It usually takes a few good shakes of the bottle to overcome the sauce’s Yield stress and to make it run. On an industrial scale, another Bingham fluid is “red mud” tailings in the Bayer alumina-from-bauxite production process.

In Fig.5.1 we have two Newtonian fluids 1 and 2 whose dynamic viscosities are \( \mu_1 \) and \( \mu_2 \) respectively. The lines intersect the Bingham fluid line at points 1 and 2 and so we can say that the Bingham fluid has two apparent viscosities \( \mu_{a1} \) and \( \mu_{a2} \) identical in value to \( \mu_1 \) and \( \mu_2 \). We can similarly assign an infinite number of apparent viscosities on a Bingham fluid line but we need only two points (and not less than two) to define the slope \( \eta \).

Shear stress \( \tau \) of a flowing Bingham fluid in a pipe varies from \( \tau_w \) at the wall to zero at the centreline, just like in Newtonian fluids. Solution of the differential equation 5.1 yields a parabolic velocity distribution in an annulus between the pipe wall and an inner radius \( r_0 \), at which \( \tau = \tau_w \) as shown in Fig.5.2. The maximum local velocity \( v_s \) is at \( r_0 \) and the whole central plug also flows with this velocity. The average velocity of flow is given by \( V=D\tau_w/(8\eta)[1-4/3(\tau_0/\tau_w)+1/3(\tau_0/\tau_w)^4] \), which was developed by Buckingham. The ratio \( v_s/\tau_w \) is always less than 1 and \( (\tau_0/\tau_w)^4 \) is that much smaller. By neglecting this term and rearranging the remaining terms we get:

\[
\tau_w = 4/3.\tau_0 + \eta.8V/D \quad (5.2)
\]

If we let \( dv/dy=8V/D \), equation 5.2 produces a line (marked asymptote) parallel to the line of equation 5.1, but \( \tau_0/3 \) higher up.

The Buckingham equation produces a true curve of the fluid (shown dotted) between these two lines but this equation cannot be used to find \( \tau_0 \) and \( \eta \) because it already contains them. To find \( \tau_0 \) and \( \eta \) we must use either
equation 5.1 or 5.2. Both yield reasonable solutions and we might as well select the simpler of the two, namely 5.1. We make \( \frac{dv}{dy} = \frac{8V}{D} \) in the Shear stress graph of Fig.5.1, which then becomes the Pseudo shear stress graph. Test points usually fall on a straight line and it is a simple matter then to measure its slope \( \eta \) and its vertical intercept \( \tau_o \).

We therefore modify equation 5.1 for Bingham fluids to:

\[
\tau_w = \tau_o + \frac{\eta}{8} \frac{V}{D} \quad (5.3)
\]

Next we use equation 5.3 and Fig.5.1 to derive an expression for the apparent dynamic viscosity:

\[
\mu_a = \eta + \frac{\tau_o}{8 \frac{V}{D}} \quad (5.4)
\]

and substitute it in the equation for Reynolds number \( N_R = \rho m \frac{D V}{\mu_a} \) which we need to calculate \( V_c \), the critical average velocity at the end of the laminar flow range. This usually occurs at a Reynolds number of 2000. After some substitutions and manipulation of terms, we finally get:

\[
V_c = X_1 + \sqrt{(X_1^2 + X_2)} \quad \text{where: } \quad X_1 = \frac{\eta N_R}{2 \rho m D} \quad \text{and: } \quad X_2 = \frac{\tau_o N_R}{8 \rho m} \quad (5.5)
\]

At velocities lower than \( V_c \), flow is laminar, as we have seen, and total head \( H_m \) varies relatively little with changes of flow rate \( Q \). Pumping costs are therefore approximately directly proportional to flow rate. At velocities higher than \( V_c \), flow is turbulent and head and pumping costs vary parabolically i.e. proportional to flow rate squared. The most economical pumping velocity is therefore the one nearest to \( V_c \).

**EXAMPLE: ES5.1**

Pumping Bingham Slurries

Slurries of water and finely ground limestone are used in the production of cement. These slurries are usually of Bingham type. At one cement plant they needed to know the lowest-cost flow rates for various pipe diameters and slurry densities. A pilot plant was built comprising a slurry pump, a holding tank, flow control and measuring equipment, instrumentation and two 100 m long pipes of \( D=0.150 \) m and \( D=0.200 \) m diameter as well as return pipes and accessories. The static head was equal to zero.

The solids were limestone (88%) and clay (12%) with an overall density of \( \rho = 2650 \) kg/m\(^3\). The mass screen analysis and the \( d_{50} \) particle size are shown in Table 5.1 and particle size distribution is plotted in Fig.5.3:

<table>
<thead>
<tr>
<th>Sieve opening size (( \mu m ))</th>
<th>51</th>
<th>88</th>
<th>149</th>
<th>297</th>
<th>841</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative passing (%)</td>
<td>78</td>
<td>89</td>
<td>96</td>
<td>99</td>
<td>100</td>
</tr>
</tbody>
</table>

Slurries were mixed and prepared on site to the required concentrations between \( C_w = 50 \) and 65% in 5% increments. Flow rates, pressure losses, slurry temperatures and \( C_w \) were recorded.

Hundreds of test points were collected and plotted and curves of constant \( C_w \) were fitted through all the points (not shown here). Here we deal only with the slurry of concentration \( C_w = 65\% \) (ie. density \( \rho = 1680 \) kg/m\(^3\)). A few points from the tests are listed in Tables 5.2 and 5.3 on the next page.

The \( Q, H_m \) test points were plotted in Fig.5.4 in which the shallow-angled portions represent laminar flow and the steeper portions turbulent flow. Water friction curves for the same pipes are also shown for comparison. They were obtained by means of the Darcy diagram (see Section 6).

Tables 5.2 and 5.3 also show derived values \( 8V/D \) and \( \tau_w \) \([=PD/(4L)=g.\rho.Hm.D/(4L)]\). These points were plotted in Fig.5.5.

**LAMINAR FLOW:** We can now calculate the slope of the coefficient of rigidity \( \eta \) from the test values and by reference to Fig.5.5. The right-most and left-most points on the laminar
flow line are point 3 of the 0.150 m pipe (ie point 3.150) and point 1 of the 0.200 m pipe (i.e. point 1.200). The coefficient of rigidity is then given by:

\[
\eta = \frac{(\tau_{w3.150} - \tau_{w1.200})}{(8V/D_{3.150} - 8V/D_{1.200})} = \frac{(23.24 - 20.80)}{(99.2 - 35.5)} = 0.0383 \text{ Pa.s}
\]

We get the Yield stress \(\tau_o\), by means of equation 5.2 from any point on the laminar flow line, say point 2 of the 0.200 m pipe:

\[
\tau_o = \tau_{w2.200} - \eta \cdot 8V/D_{2.200} = 21.40 - 0.0383 \times 51.2 = 19.44 \text{ Pa}
\]

We must finally obtain the critical pipeline velocities \(V_c\) for the required pipe diameters. Critical velocities were calculated from equation 5.5 for Reynolds numbers 2000. They are listed in Table 5.4 together with corresponding critical flow rates \(Q_c\).

Comparison of these calculated \(V_c\) values with actual \(V\) test values at the end of the laminar ranges – see points 3 for the 0.150 m and 0.200 m pipes in Tables 5.2 and 5.3 – shows that the calculated values are very close to the test values. This confirms also that our selection of equation 5.3 is reasonably correct. In conclusion, we are fairly safe to assume that the \(V_c\) values for the 0.100 m and 0.250 m pipes are also reasonably accurate and that the flow rates tabulated in Table 5.4 would yield the most cost effective pumping rates for the given slurry in the four pipes. [Slurry system resistance for \(D=0.15m\) in Fig.5.4 is used in Fig.10.1 with \(Z>0\)].

### TURBULENT FLOW

Having obtained the values of \(V_c\) for the two pipes and having checked and plotted \(H_m\) and \(Q\) value pairs for laminar flow, we can also check and plot reasonably accurate pairs of \(H_m\) and \(Q\) for turbulent flow.

In turbulent flow, pipe head losses \(H_f\) are, by Darcy’s equation (see Section 6), proportional to velocities \(V\) squared, ie \(H_f = KxV^2\), which is the equation of a parabola and \(K\) is a constant of proportionality. If we take any two points 1 and 2 on the parabola, we can eliminate \(K\) if we write \(H_{f1}/V_1^2 = H_{f2}/V_2^2\), from which we extract \(H_{f2} = H_{f1}(V_2^2/V_1^2)\).

Critical velocities \(V_c\) belong to both laminar and turbulent flows and correspond to points 3 in Tables 5.2 and 5.3 and in Fig 5.4. We can therefore write \(H_{fc} = H_{f1}(V_2/V_1)^2\) and substitute \(H_{fc}\), \(V_c\) and \(V_1\) values from the two tables. This yields \(H_{fc} = 6.16\) m for the 0.150 m pipe and \(H_{fc} = 4.05\) m for the 0.200 m pipe. Both these values are within 5% of the \(H_m\) values in the two tables, which is close enough for most practical purposes.

### Table 5.2 – Tests with \(D=0.150\) m Pipe

<table>
<thead>
<tr>
<th>Point</th>
<th>(V) [m/s]</th>
<th>(Q) [L/s]</th>
<th>(H_m) [m slurry]</th>
<th>(8V/D) [s(^{-1})]</th>
<th>(\tau_w) [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.67</td>
<td>12</td>
<td>3.37</td>
<td>35.5</td>
<td>20.80</td>
</tr>
<tr>
<td>2</td>
<td>1.06</td>
<td>19</td>
<td>5.30</td>
<td>56.4</td>
<td>21.60</td>
</tr>
<tr>
<td>3</td>
<td>1.86</td>
<td>33</td>
<td>5.76</td>
<td>99.2</td>
<td>23.24</td>
</tr>
<tr>
<td>4</td>
<td>2.38</td>
<td>42</td>
<td>5.85</td>
<td>126.8</td>
<td>33.20</td>
</tr>
</tbody>
</table>

### Table 5.3 – Tests with \(D=0.200\) m Pipe

<table>
<thead>
<tr>
<th>Point</th>
<th>(V) [m/s]</th>
<th>(Q) [L/s]</th>
<th>(H_m) [m slurry]</th>
<th>(8V/D) [s(^{-1})]</th>
<th>(\tau_w) [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.68</td>
<td>21</td>
<td>2.49</td>
<td>26.2</td>
<td>20.48</td>
</tr>
<tr>
<td>2</td>
<td>1.28</td>
<td>40</td>
<td>2.60</td>
<td>51.2</td>
<td>21.40</td>
</tr>
<tr>
<td>3</td>
<td>1.82</td>
<td>57</td>
<td>2.70</td>
<td>72.8</td>
<td>22.23</td>
</tr>
<tr>
<td>4</td>
<td>2.23</td>
<td>70</td>
<td>3.86</td>
<td>89.1</td>
<td>31.80</td>
</tr>
</tbody>
</table>

### Table 5.4 – Economic Pipeline Flow rates

<table>
<thead>
<tr>
<th>(D) [m]</th>
<th>(V_c) [m/s]</th>
<th>(Q_c) [L/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.100</td>
<td>1.94</td>
<td>15</td>
</tr>
<tr>
<td>0.150</td>
<td>1.87</td>
<td>33</td>
</tr>
<tr>
<td>0.200</td>
<td>1.81</td>
<td>57</td>
</tr>
<tr>
<td>0.250</td>
<td>1.79</td>
<td>88</td>
</tr>
</tbody>
</table>

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We could finally redraw the whole $H_f$ vs $Q$ resistance curves for both pipes and they would look very much like those we started from in Fig.5.4. We could similarly calculate other points from 3 to 4 and beyond, and for other pipe sizes.

**VISCOMETERS**

Rheological parameters $\tau_o$ and $\eta$ of Bingham fluids are often obtained from tests with a suitable viscometer. There are at least two such commercially available units, the Stormer Viscometer and the DeVaney-Shelton Consistometer. Other, simpler ones can be built in-house.

**Stormer Viscometer**

This unit has a vertical cup within which another cup is held and rotated concentrically by a motor. Slurry is poured between the cups, the inner cup is rotated at a selected speed and the resisting torque is measured on the outer cup. The test is repeated at various other selected speeds.

**DeVaney-Shelton Consistometer**

This unit has a vertical cylinder connected through an inverted bellmouth to a concentric vertical capillary tube at the bottom. The cylinder has internal vertical baffles and a centrally-mounted motor-driven agitator, to prevent solids settling. By closing the capillary with a finger, the agitator in put in motion at a selected speed and a known volume of slurry is put in the cylinder. When the finger is removed, the emptying period is timed.

The main disadvantage of both these units is that the test results are not directly comparable to flow regimes in pipes. The readings do not yield $\tau_o$ and $\eta$ directly. The results must be carefully interpreted to make them relevant to actual pipeline slurry pumping.

**Tube viscometer**

This is a much more direct unit that needs no calibration and one that can be built reasonably economically with few basic tools. It consists basically of (1) a vertical clear glass or plastic cylinder with a horizontal outlet at the bottom, (2) a straight horizontal stainless tube, (3) a rubber hose to connect 1 and 2 and, (4) a rubber pinch valve at the tube’s end. Some gentle agitation may be required if the slurry tends to settle.

Typical dimensions are as follows:

The vertical cylinder is from 50 to 100 mm inside diameter and some 500 mm long, the tube is 5 to 10 mm bore (D) and is 140D long. There is a first tapping point at 20D downstream from the hose. This section acts as a flow-straightener. The second tapping point is 100D downstream from the first. This is the test section of length L. The last 20D act as a flow steadier. The tapping points are 2 mm diameter vertical holes, drilled and deburred to minimise turbulence. Short nipples are carefully brazed over the holes and clear-plastic vertical tubes with open ends are fitted to the nipples. During testing, a small amount of water may be dribbled as needed into the open tops to wash down any accumulated slurry and improve visibility for head measurements.

Each flow rate is set with the pinch valve and, when the flow steadies, $Q$ is obtained by timing the filling of a container of known volume. Static slurry heads $H_m$ are measured at the same time on both plastic tubes and head losses $H_f$ are then calculated from the differences of the two heads. The flow is changed and the test repeated four or five times.

If the elevation of the cylinder above the tube is not sufficient, a tight cap is usually fitted to the open top of the cylinder. The cap is connected to a source of compressed air at known (but variable) pressures $P$ to provide the driving heads $H_m$ for the slurry flows.

What happens in this viscometer is an exact replica of what happens in an actual pipeline of larger diameters and longer lengths. The results are completely convertible without any calibrations or special knowledge.

The tube viscometer can be readily dismantled for storage and quickly reassembled when required. It is particularly handy when only a limited amount of slurry is available for testing.

Another, simplified construction of tube viscometer, employs the same vertical cylinder, rubber hose connectors and pinch valve, but instead of one horizontal tube it uses two, one of 50D and the other 100D. There are no tapping points in these tubes. Heads are taken directly from the vertical cylinder. Test are run as described above but with each tube. Test readings from the two runs are plotted and curves drawn through them on the same graph. Since entrance and exit losses for each pipe are the same for each flow rate, we then read the difference between the two curves off the graph at various flows and replot them. A new curve is then drawn through these points. This curve represents the true head losses for the difference of pipe lengths.
SLUMP PLATE

Although not really a viscometer, this portable unit gives a quick visual indication of whether a slurry is potentially pumpable with centrifugal pumps. It consists of a 300 mm square by 2 mm thick stainless flat plate with concentric rings inscribed in one surface. The central ring is 50 mm diameter and the other diameters are progressively larger by 20 mm. A short thin pipe 50 mm bore by 50 mm long with machined and squared ends is placed in the middle of the plate and a sample of slurry is poured into it. The pipe is then gently removed vertically to allow the slurry to spread (or slump) out evenly all around. If the slurry does not spread out to at least the third ring it is too thick and a centrifugal pump will usually not be able to pump it.
SECTION 6 – Friction Losses in Pipes

PUMPING WATER
Whenever energy is expended to do useful work, there is always some additional energy being wasted through friction and heat. Under normal conditions, the effects of friction can be as visually impressive as the fireball produced by a spacecraft re-entering the atmosphere, or they can be hardly detectable like water getting warmer when passing through an operating pump.

Liquid passing through a pipe is likewise subjected to internal friction and energy loss. It is necessary, when designing pipeline systems, to predict the magnitude of the loss and provide sufficient power to the liquid at the beginning of the pipe so that the required flow rate will be delivered at the end. Darcy, Weisbach and others proposed the following equation that, over time, has become the virtual standard:

\[ H_f = f \cdot L \cdot \frac{V^2}{2gD} \]  

Darcy friction factors have been obtained experimentally on many liquids and many pipe surface roughnesses by various researchers. Their magnitude depends on (1) \( N_R \), the Reynolds Number of the medium in the pipe (see Section 4) and (2) the ratio \( e/D \), the Relative roughness of the pipe in which \( e \) is the average height of projections on the pipe wall. The relationship between \( f \), \( N_R \) and \( e/D \) is usually plotted on a logarithmic diagram (see Diagram A1–1 in Appendix A). Churchill developed an equation, which is too cumbersome for calculating occasional, single values of \( f \), but is tremendously helpful when plotting the whole Darcy diagram, including laminar, transition and turbulent flow regions, say from \( N_R=10^3 \) to \( 10^8 \) and from \( e/D=0 \) to 0.01 yielding \( f \)-values from \( f=0.01 \) to 0.04. The Churchill equation is particularly handy in spreadsheets and in computer programs:

\[ f = g \left\{ \left( \frac{8}{N_R} \right)^{12} + \left[ \frac{3750}{N_R} \right]^{16} \left\{ -2.457 + \ln \left[ \left( \frac{2}{N_R} \right)^{0.9} + 0.27 \frac{e}{D} \right] \right\}^{36} \right\}^{1/12} \]  

A small but interesting portion of the full Darcy diagram is shown Fig.6.1 on the next page. Three rhomboidal areas in it represent commonly used pipe materials – basalt, steel and plastics – in decreasing order of height projections \( e \). Pipe diameters \( D=0.050 \) m and 0.500 m as well as pipe velocities \( V=0.5 \) m/s and 5.0 m/s define the boundaries of common applications within each area. Taken together, the three areas represent the part of the full diagram that covers most of the practical duties for water and slurries in turbulent flow. For duties outside of this diagram, or for greater accuracy, use the above equation or the full Darcy diagram.

Hazen and Williams devised another method for predicting pipe friction losses. They tested many pipes with water at 20°C, analysed the results and allocated friction factors \( C \) from 80 (for rough, corroded pipes) to 160 (for smooth pipes). These factors \( C \) are shown in the same Fig.6.1 for a direct comparison with the Darcy \( f \) factors. A full Hazen-Williams Diagram A1-2 for various \( D \), \( V \) and \( C \) factors is given in Appendix A1. The Hazen and Williams equation in metric format, is:

\[ V = 0.35422D^{0.63} C \left( \frac{H_f}{L} \right)^{0.54} \]  

After substituting equation 6.3 in 6.1, rearranging terms and contracting some of the terms into \( N_R \), we get the following equality:

\[ C = 43.67 / (f^{0.54} \cdot N_R^{0.081}) \]  

Values of \( f \) and \( N_R \) for the vertical midpoints of the three rhomboids in Fig.6.1 were substituted in equation 6.4. The resulting \( C \) values are listed in Table 6.1. These values may be used to obtain pipe friction losses from the Hazen-Williams diagram, but only for liquids with the same viscosity as water at 20°C.

<table>
<thead>
<tr>
<th>Pipe material</th>
<th>C factor for water at 20°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basalt</td>
<td>110 – 140</td>
</tr>
<tr>
<td>Steel</td>
<td>130 – 150</td>
</tr>
<tr>
<td>Plastics</td>
<td>150 – 160</td>
</tr>
</tbody>
</table>
PUMPING SLURRIES

There are various methods for calculating friction losses when pumping slurries, some more complex than others. The one thing to keep in mind however is that, because of the infinite number of different combinations of particle sizes, which can occur in slurries, it is very difficult to predict correct friction losses by any means. However, the method shown here produces values, which are good enough for short pipelines. More rigorous treatment – and certainly some test work – is required for long pipelines.

The great majority of solids pumping applications occur in mining and mineral processing plants where pipelines are relatively short, say up to a few hundred metres and static heads are also not excessive, say up to 30 metres. It is usual in such cases to use some simplified methods for predicting friction losses. In most cases these calculations produce results that are sufficiently close, say within 5%, to the actual requirements.

For longer pipelines or for unusual applications, it would take a very brave and experienced engineer and a very trusting plant owner/operator to approve the construction of an expensive pipeline based only on calculations. In all critical cases some amount of test work is absolutely necessary.

Friction losses are basically of three types, namely when pumping: (1) liquids, (2) homogeneous slurries of fine non-settling solids and (3) heterogeneous slurries of larger (settling) solids.

Liquids (1) are handled by the Darcy or Hazen-Williams methods. Non-settling slurries (2) are taken care of by the Bingham slurry method. Settling slurries (3) are described next.

The first thing we have to determine is the limiting settling velocity \( V_L \) for the required mass flow \( M \) of solids. Pumping solids at high concentrations \( C_w \) requires small flow rates \( Q \) and small pipe diameters \( D \), and the

\[
\text{Valid for water at } 20^\circ\text{C only}
\]

**Fig.6.1 – Comparison of Darcy and Hazen-Williams friction factors**
opposite holds with low concentrations. Usually only one combination will satisfy the required \( V_L \). This is done by means of the Durand method, or by Wilson’s if preferred, or by test work.

To use Durand, proceed as follows:

1. For a fixed solids mass flow \( M \), select a value of \( C_v \) and calculate the corresponding \( Q \).
2. Select a standard pipe of inner diameter \( D \).
3. Obtain from the Durand diagram the corresponding \( V_L \) and multiply it by the pipe cross-section to obtain the limiting flow rate \( Q_L \).
4. If \( Q \) is some 10 to 15% larger than \( Q_L \), the pipe diameter is correct and the solids will not settle during pumping.
5. If \( Q \) is equal to or smaller than \( Q_L \), select the next smaller pipe diameter and return to step 3.
6. If no reasonable pipe diameter satisfies the set requirements, select another value of \( C_v \) and return to step 1.

The slurry friction head (in metres of slurry) will be numerically larger than the water friction head (in metres of water) at the same flow rate \( Q \).

We now have a starting point (see Fig.6.2). Static Head \( Z \) is here assumed to be zero. If \( Z \) is positive (or negative), it must be drawn as a straight line, parallel to the bottom of the graph at a distance \( Z \) above (or below) the base. The left point of the water resistance curve then starts from this \( Z \) line instead of from zero.

We first calculate water pipe friction loss (m water) from the Darcy diagram at 3 flow rates, plot the points and draw the water resistance curve through them. Next we mark \( Q_i \) on the base line, draw a vertical line to the water curve at \( a \) and from there a horizontal line leftwards. We next draw two vertical lines up from the base: one from 0.7\( Q_i \) to meet the horizontal line at \( b \), the other from 1.3\( Q_i \) to the water curve at \( c \). Finally, we draw a square parabola with vertex at \( b \) and tangential to the water curve at \( c \). This is the slurry resistance curve. Now we mark \( Q \) on the base line and draw a vertical line to intersect the slurry resistance line at \( d \) and from there a horizontal line to the left axis at \( e \), which gives the friction head loss \( H_f \) (m of slurry).

**DREDGING**

Dredging is a whole field to itself and it would justify its own Manual. In dredging, particle size distribution and solids concentrations vary continuously and so do limiting settling velocities. There is therefore not much sense using any complicated methods for predicting friction head losses. By skilfully manoeuvring the dredge and its suction pipe (below the water surface) into dredging deposits and out of them, a good dredge master can ensure that the dredge operates at its top mass flow capacity without blocking the pipeline.

The Colorado School of Mines developed a simple method for predicting slurry friction heads in dredging pipelines, based on losses when pumping water. First draw the system resistance curve for water (m water) as explained above. Next select a factor from Table 6.2, corresponding to the worst expected dredged material. Then multiply water heads at various flow rates by this factor and draw the slurry friction curve (m slurry). Finally add the static head to the friction curve to get the total system resistance curve \( H_{m-Q} \), as usual.

<table>
<thead>
<tr>
<th>Worst type of material dredged</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light silt, mud or silt, but no sand</td>
<td>1.10</td>
</tr>
<tr>
<td>Mud, fine sand or soft clay</td>
<td>1.15</td>
</tr>
<tr>
<td>Medium sand, mud and clay mixed</td>
<td>1.20</td>
</tr>
<tr>
<td>Tough clay, coarse sand and/or gravel</td>
<td>1.30</td>
</tr>
<tr>
<td>Coral or shell</td>
<td>1.40</td>
</tr>
<tr>
<td>Coarse gravel and boulders without clay</td>
<td>1.50</td>
</tr>
</tbody>
</table>
SECTION 7 – Solids Settling in Slurries

We are all aware of the awesome power of fluids, particularly air and water, when they move at high velocities. Storms, cyclones, tsunamis, floods and similar quirks of nature can uproot trees, destroy buildings, cause landslides, change coastlines and generally produce massive devastation. On a much smaller scale we can use the same forces of nature for the controlled transportation of solids through pipes.

Water flowing around solid particles creates pressure differentials around them and the resulting drag forces move the particles in the general direction of the flow. The velocity of the solids is slower than that of the water. This is called slippage and particles of different sizes and densities have different slippages. Relative to horizontal flow, slippage increases in uphill flow and decreases in downhill flow because gravity slows down and respectively accelerates the flow of solids relative to the liquid. It follows that, in various parts of a convoluted pipeline, local solids concentrations vary, which influences local slurry velocities, pipe wear and friction losses.

In any pipeline such local variations can cause solids to settle and possibly to block the pipe. Flow conditions must therefore be fully investigated and often confirmed by means of some test work. For short pipelines (up to a few 100 m) on pumping duties, with known materials in typical mining pumping circuits this is not required. We base all calculations only on average solids concentrations (both Cw and Cv) as they exist at the end of a pipeline.

When the sizing, density and concentration of solids in a slurry are known, we must determine the average Limiting settling velocity $V_L$ of the slurry, which will move the solids and not let them settle.

Durand and Condolios carried out most of the original investigative work in the 1950s. They worked with water and narrowly graded solids i.e. the $d_{50}$ particles were less than twice the size of the $d_{20}$ particles. They produced the graph shown in Fig.7.1 and the following formula:

$$V_L = F_L \sqrt{2gD(S-1)} \quad (7.1)$$

The diagram and formula are still in wide use today. Diagram A1-5 with nomogram, which gives $V_L$ without calculations is given in Appendix A1.
Where the solids have a good spread of sizes, i.e. the \( d_{80} \) particles are more than 5 times the size of the \( d_{20} \) particles, Cave’s Modified limiting settling velocity diagram shown in Fig. 7.2 gives more accurate predictions.

In either case the particles smaller than 100 \( \mu m \) combine with the carrier liquid to form a heavier carrier liquid. It is always prudent to make the actual pumping velocity some 10% higher than the calculated \( V_L \) to cater for uncertainties and possible changes of future pumping conditions.

Tests and studies to predict more accurate limiting settling velocities are being carried out by researchers more or less continuously. Some of the more prominent contributions from the 1970s to the present were due to K. C. Wilson who introduced a Two-Layer model of flow of liquid and settling solids in a pipe. A brief and simplified description of this method is as follows:

In a pipe with slurry at rest, all solids are settled at the bottom of the pipe and the liquid is at the top. As pumping starts and water velocity increases, water picks up progressively more solids until, at a certain velocity \( V_c \) (at any solids concentration) the last solids at the bottom of the pipe are on the point between moving and staying put. Wilson produced a graph to estimate these deposition velocities \( V_c \) as a function of pipe diameter, particle size and solids density. The results are applicable to the highest pumpable solids concentrations. Diagram A1-6 similar to Wilson’s is in the Addenda A1.

A comparison of Durand’s \( V_L \) and Wilson’s \( V_c \) has been made here for solids of two specific gravities (\( S=2 \) and \( S=4 \)), three different particle sizes \( (d_{50}=0.15, 0.5 \) and \( 10 \) mm) to be pumped at \( C_v=15\% \), through pipes of three different diameters \( (D=0.1, 0.2 \) and \( 0.4 \) m). The results are shown in Fig. 7.3 (for \( S=2 \)) and Fig. 7.4 (for \( S=4 \)).

The figures show that Durand and Wilson agree that maximum deposition velocities occur with particle sizes of around 0.5 mm and that depositions start at lower velocities with particles, which are either smaller or larger than 0.5 mm.

The figures also show that Durand’s \( V_L \) values are in all cases considerably higher than Wilson’s \( V_c \) values. It is possible that Wilson used more modern instrumentation or more careful observation, but it is also possible that Wilson and Durand simply used different criteria for what Wilson calls \( V_c \) a “velocity at the limit of stationary deposition” and Durand calls \( V_L \) a “limiting settling velocity”. Whatever the reasons for these discrepancies, they are beyond the scope of this short Manual. All that can be said here is that for quick and ready designs of slurry pipelines, the Durand method yields higher pumping velocities than Wilson’s and therefore it has more built-in safety against pipe blockages.

On the other hand, if Wilson’s lower deposit velocity predictions are correct and larger pipes are selected – here we are expressing only caution, not doubt – the great advantage over Durand’s smaller pipes and higher velocities would be less pipe wear and lower friction losses with considerable savings in pumping costs.

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In conclusion, we can say that the Durand method has been used over a long period, but it is possible that it is overly conservative. For this reason Wilson’s method should be considered, but it should be applied with caution until sufficient confidence is accumulated with experience.

EXAMPLE: ES7.1

Fine particles mixed with water make ‘heavy liquid’

Very fine solid particles – usually below 100 µm in diameter – do not settle in slurries. They remain in suspension and for all practical purposes they become part of the carrier liquid. They may affect the liquid density and viscosity and the slurry’s limiting settling velocity. Let us consider pumping solids of S=3.1 with water as the carrier liquid in a slurry with a solids concentration of Cw=46% in a pipe of D=0.150 m.

Particle size distribution of solids collected on progressively smaller screen openings is shown in Table 7.1 and plotted in Fig.7.5. The median particle size is \( d_{50} = 190 \) µm. The ratio \( d_{80}/d_{20} = 500/70 = 7.1 \) is greater than 5 so we use the Cave’s Modified Limiting Settling Velocity Graph from Fig.7.2, which gives \( F_{L1} = 1.05 \) and \( V_{L1} = 2.61 \) m/s.

All relevant calculations are shown in Table 7.2 using mostly equations from Table 2.1. From Fig.7.5 we estimate that 29% of solids by mass is smaller than 100 µm in the total solids.

Fig.7.6 shows particle size distribution when all these –100 µm fine solids are removed from the total solids and they are

<table>
<thead>
<tr>
<th>Particle sizes (µm)</th>
<th>Mass retained</th>
</tr>
</thead>
<tbody>
<tr>
<td>-88</td>
<td>25%</td>
</tr>
<tr>
<td>-105 +88</td>
<td>5%</td>
</tr>
<tr>
<td>-250 +105</td>
<td>30%</td>
</tr>
<tr>
<td>-500 +250</td>
<td>20%</td>
</tr>
<tr>
<td>-1000 +500</td>
<td>20%</td>
</tr>
</tbody>
</table>

Table 7.1 – Particle size distribution of sample

<table>
<thead>
<tr>
<th>Particle sizes (µm)</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>-88</td>
<td>25%</td>
</tr>
<tr>
<td>-105 +88</td>
<td>5%</td>
</tr>
<tr>
<td>-250 +105</td>
<td>30%</td>
</tr>
<tr>
<td>-500 +250</td>
<td>20%</td>
</tr>
<tr>
<td>-1000 +500</td>
<td>20%</td>
</tr>
</tbody>
</table>

Table 7.2 – Calculations with water

<table>
<thead>
<tr>
<th>S</th>
<th>Sw =1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cw</td>
<td>=46%</td>
</tr>
<tr>
<td>Cv</td>
<td>=21.5% =1 / (1–3.1+3.1/0.46)</td>
</tr>
<tr>
<td>Sm</td>
<td>=1.45 =1 / [1–0.46(1–1/3.1)]</td>
</tr>
<tr>
<td>Mm</td>
<td>=1kg</td>
</tr>
<tr>
<td>Ms</td>
<td>=0.460kg =0.46x1</td>
</tr>
<tr>
<td>VOLs</td>
<td>=0.148L = 0.46 / 3.1</td>
</tr>
<tr>
<td>Mw</td>
<td>=0.540kg = 1–0.460</td>
</tr>
<tr>
<td>VOLw</td>
<td>=0.540L =0.540 / 1</td>
</tr>
<tr>
<td>VOLm</td>
<td>=0.688L =0.148+0.540</td>
</tr>
<tr>
<td>F L1</td>
<td>=1.05</td>
</tr>
<tr>
<td>V L1</td>
<td>=2.61m/s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S</th>
<th>Sw =1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cw</td>
<td>=46%</td>
</tr>
<tr>
<td>Cv</td>
<td>=21.5% =1 / (1–3.1+3.1/0.46)</td>
</tr>
<tr>
<td>Sm</td>
<td>=1.45 =1 / [1–0.46(1–1/3.1)]</td>
</tr>
<tr>
<td>Mm</td>
<td>=1kg</td>
</tr>
<tr>
<td>Ms</td>
<td>=0.460kg =0.46x1</td>
</tr>
<tr>
<td>VOLs</td>
<td>=0.148L = 0.46 / 3.1</td>
</tr>
<tr>
<td>Mw</td>
<td>=0.540kg = 1–0.460</td>
</tr>
<tr>
<td>VOLw</td>
<td>=0.540L =0.540 / 1</td>
</tr>
<tr>
<td>VOLm</td>
<td>=0.688L =0.148+0.540</td>
</tr>
<tr>
<td>F L1</td>
<td>=1.05</td>
</tr>
<tr>
<td>V L1</td>
<td>=2.61m/s</td>
</tr>
</tbody>
</table>

Table 7.3 – Calculations with heavy liquid

<table>
<thead>
<tr>
<th>S, Sm, Mm and VOLm are unchanged</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cf</td>
</tr>
<tr>
<td>Mf</td>
</tr>
<tr>
<td>VOLf</td>
</tr>
<tr>
<td>Mc</td>
</tr>
<tr>
<td>VOLc</td>
</tr>
<tr>
<td>Mw'</td>
</tr>
<tr>
<td>VOLw'</td>
</tr>
<tr>
<td>Sw'</td>
</tr>
<tr>
<td>Cv'</td>
</tr>
<tr>
<td>Cv'</td>
</tr>
<tr>
<td>FL2</td>
</tr>
<tr>
<td>V L2</td>
</tr>
</tbody>
</table>

=1.1\sqrt{2x9.81x0.15(3.1/1.154 – 1)}
combined with water to form a heavy liquid. The median particle size of the remaining coarse solids is now \( d_{50} = 280 \mu m \). The ratio \( d_{80}/d_{20} = 670/160 = 4.2 \) is closer to 5 than 2 so we use Fig. 7.2 once more. This time \( F_{1.2} = 1.1 \) and \( V_{L2} = 2.45 \text{ m/s} \).

Calculations for this alternative are shown in Table 7.3 on the previous page.

Velocity \( V_{L2} \) is 6.5% slower than \( V_{L1} \) and since pipe friction is proportional to velocity squared, friction loss in the second case would be 13% smaller if we based our actual velocity on \( V_{L2} \). Actual pipeline velocity should always be made at least 10% higher than the calculated \( V_L \) to cater for uncertainties and future duty changes. We should therefore recommend a pumping velocity \( V = 2.7 \text{ m/s} (=1.1 \times 2.45) \).

**NOTE:**
The heavier liquid can carry the coarse solids with a slower \( V_L \) because it provides more buoyancy for them. However, these same coarse solids have a larger \( d_{50} \), which might require a faster \( V_L \). These two corrections work against each other and therefore, depending on the magnitudes of the respective corrections, they might occasionally cancel out or we might get a \( V_{L2} \) higher than \( V_{L1} \) instead of the desired opposite. In this case we pick the higher of the two.
Cavitation is a phenomenon that adversely affects the performance of a centrifugal pump and it must be avoided during normal operation. The onset of cavitation in a pump, at any given speed and flow rate, is brought about by a particular combination of temperature and pressure at the pump suction flange. The absolute total head is called the Net Positive Suction Head or NPSH. The letter P tells us that NPSH, by definition, can never be a negative number.

Vapour Pressure is the pressure acting on a body of liquid – in slurry pumping mostly water – at which the liquid boils at a particular temperature. By varying the pressure, we can make the liquid boil at virtually any temperature: the lower the pressure, the lower the boiling temperature. This explains why at altitudes, high above sea level, water boils at below 100ºC and food takes longer to cook.

At atmospheric pressure and on the point of boiling, tiny spheres of water convert to vapour bubbles thereby expanding their original volumes 1600 times. If the vapour bubbles then move to a zone of higher pressure, they immediately implode, with considerable force, back to their original liquid volumes. In an open vessel, all these implosions simply dissipate quietly through the boiling liquid surface into the surrounding ambient. In a closed vessel on the other hand, the implosions generate loud, localised pressure shocks, which cause intermolecular cracks on internal metallic surfaces, gradually dislodging small solid particles and finishing up with sponge-like cavities. This is cavitation damage and the process causing it is cavitation. This type of damage does not usually occur on rubber surfaces because there are no inter-crystalline boundaries and rubbers simply absorb the shocks.

Water passing through a centrifugal pump is similarly subjected to low and high pressure zones. The lowest pressure exists at the eye of the impeller. If this pressure falls below the vapour pressure, local boiling takes place, generating masses of tiny vapour bubbles within the liquid just past the leading edges of the pumping vanes. These bubbles implode and can cause damage as soon as they are swept downstream to zones of higher pressures – only to be replaced immediately with new ones.

The continuous procession of new vapour bubbles produces what appears to be a stationary cloud of vapour at the impeller eye, throttling the flow of water. The end effect is a drop in flow rate Q and of total head H and a reduction in pump performance, which is – as much as any surface damage – the reason why a pump should operate under conditions sufficiently free from cavitation.

It is standard practice to test every centrifugal pump on water in order to produce sets of Total Head vs Flow rate and Efficiency vs Flow rate curves at minimum and maximum design speeds and some intermediate speeds. Tests are also carried out to determine the lowest NPSH required by the pump for cavitation-free performance or NPSHr.

Fig.8.1 shows a typical pump test setup for determining NPSHr from the available NPSH, or NPSHa.
NPSHa = Hatm – Hvap + Hgs + Hvs \hspace{1cm} (8.1)

Where:
Hatm = Atmospheric head (from Fig.8.3) [m water]
Hvap = Vapour head of water (from Fig.8.3) [m water]
Hgs = Gauge suction head – positive (+) in Fig.8.1 [m water]
Hvs = Velocity head in suction pipe [m]

NPSH tests are carried out as follows (see Fig.8.2):

We read the atmospheric head Hatm from a barometer and measure the water temperature T. Next we select the lowest design speed, open the suction valve fully and set a flow rate Q by adjusting the discharge valve. We measure the suction head Hs and discharge head Hd to calculate and plot H against NPSHa. Maintaining the same speed we then close the suction valve to reduce Q slightly and open the discharge valve to bring Q back to the original value and re-do the measurements and the plotting. We repeat the procedure a number of times. As Hs is reduced, NPSHa follows but H remains constant until eventually it reaches a point (point 1 in Fig.8.2) where it first rises – sometimes, due to liquid de-aeration caused by low pressure – and then drops. We have now reached the beginning of cavitation. The value of NPSHa at which H has dropped 3% below its constant value (point 2 in Fig.8.2) has been adopted by most international Pump Standards Authorities as the value of NPSHr. This means that, although there is incipient cavitation between points 1 and 2, its magnitude is too small to noticeably affect pump performance and we ignore it.

The whole test is repeated at two lower flow rates and then the next higher speed is selected and all tests done again. The process continues until the whole usable pump range has been covered. At the highest speeds and flows it is often impossible to carry out NPSH tests because of limits imposed by the test rig, namely the pump might cavitate due to insufficient NPSHa at the running conditions. Points of NPSHr are finally plotted on a set of Head vs Flow rate and Speed performance curves and lines of constant NPSHr are drawn through the points.

It is important to remember that the NPSHr values from water tests are used also when pumping most types of slurries. In other words, generally no slurry SG or other corrections are made to NPSHr values from water tests.

Additional corrections may be required when dealing with highly viscous liquids and special non-Newtonian slurries but that aspect is outside the scope of this work.

To avoid cavitation, the NPSHa on site must be always higher than the NPSHr obtained from tests. Also, NPSHa – unlike NPSHr – must take into account the density of the slurry. NPSHa must be predicted from site data and physical details as follows (see Fig.8.1):

NPSHa = Hatm – Hvap ± Zs – Hi \hspace{1cm} (8.2)
where:
Hatm = Atmospheric head [m slurry]
Hvap = Vapour head of water [m slurry]
Zs = Static suction head – shown positive (+) in Fig.8.1 [m]
Hi = Combined head loss in suction pipework [m]

It should be noted that the NPSHa values at any given speed and flow rate when pumping slurry, are always lower than when pumping water simply because the atmospheric pressure when expressed as a head of slurry is numerically smaller than when expressed as a head of water.

When selecting a pump for cavitation-free operation and when calculating NPSHa, we must not forget to make proper allowance for reduced atmospheric pressure if the pump is going to operate at high elevations Zas (say more than 1000m above sea level) and/or at high temperatures T (say above 50°C). We must furthermore ensure that NPSHa has a buffer of at least 1 metre above the corresponding NPSHr, to allow for unforeseen future duty changes.

**EXAMPLE: ES8.1**
Checking available NPSH in Mill Discharge Circuit

**DATA**
A 14/12 FF-AH rubber lined Warman pump with metal impeller will be used on a Mill Discharge duty as follows:
Plant elevation: Zas = 2000 m above sea level
Ambient temperature: T = 18°C.
Duty flow rate: Q = 500 L/s
Total Head: Hm = 35 m slurry
Head ratio: HR = 0.85 (usual for this duty)
Maximum particle size: d = 12 mm
Median particle size: d50 = 0.5 mm
Solids in slurry:
   S = 2.65
Slurry:
   Sm = 1.6, Cw = 60% by weight
Proposed sump water levels:
   Zs max = 3.5 m above pump centreline and:
   Zs min = 2.0 m
Suction pipe:
   Ds = 0.4 m dia.
   Ls = 2 m long.
   1 x 400 mm knife gate valve and
   1 x 400x350 mm reducer
Atmospheric head:
   Hatm = 8 m water (from Fig.9.3) or:
   8/1.6 = 5.0 m slurry
Vapour head:
   Hvap = 0.2 m water (from Fig.9.3) or:
   0.2/1.6 = 0.12 m slurry
Total head:
   Hw = 35/0.85 = 41.2 m water
Pump speed:
   N = 510 r/min at:
   Q = 500 and Hw = 41.2 m (from Fig.8.4)
Required NPSH:
   NPSHr = 6.5 m (from Fig.8.4)

**CALCULATIONS**
Velocity in suction pipe: Vs = 4 m/s
Velocity head: Hvs = 0.2 m
Feed sump to suction pipe inlet head loss: His = 0.5 x Hvs = 0.1 m
Equivalent pipe length of open knife gate valve: Lvs = 6 m (from graph at end of Manual)
Total equivalent pipe length: L = Ls + Lvs = 2 + 6 = 8 m
Darcy friction factor: f = 0.02 (from Darcy diagram at end of Manual)
Pipe friction loss: Hfs = f x L x Hvs / Ds = 0.02 x 8 x 0.2 / 0.4 = 0.08 m
Reducer head loss coefficient: Kr = 0.015 (from table at end of Manual)
Reducer head loss: Hrs = Kr x Hvs = 0.015 x 0.2 = 0.003 m
Combined suction head loss: Hi = His + Hfs + Hrs = 0.1 + 0.08 + 0.003 = 0.2 m

Check for cavitation: NPSHa min = Hatm – Hvap + Zsmin – Hi = 5.0 – 0.12 + 2.0 – 0.2 = 6.7 m

CONCLUSIONS

The calculated NPSHa (6.7 m) is greater than NPSHr (6.5 m) but a buffer of only 0.2 m is not safe enough. If conditions never changed, the pump would not cavitate, but conditions will inevitably vary from time to time with NPSHa dropping below NPSHr and causing cavitation. The lowest sump water level should therefore be increased by say 0.8 m to Zs min = 2.8 m to get a safer minimum NPSHa = 7.5 m.
SECTION 9 – Pump Design and Selection

PUMP TYPES
Two types of pumps are used in the hydraulic transportation of solids: centrifugal pumps and positive displacement pumps.

Centrifugal pumps are used for flow rates from a few litres to thousands of litres per second, they can handle solid particle sizes from microscopic to large rocks up to 300 mm. Their main limitation is the fact that they can develop pressures of not much more than 7 MPa even when they are arranged in series, with say up to 8 pump stages. Their casings can be of unlined or lined design, i.e. with internal replaceable liners, which can be made of many materials from soft elastomers to hard metal alloys to suit the material pumped. The wearing parts are mostly impellers, volutes and side liners.

The majority of centrifugal pumps are horizontal, i.e. they are of end-suction, horizontal-shaft design. Most wet ends can be mounted or converted into vertical pumps – used mostly as sump pumps – which are used to pump floor wash water out of floor sumps or wells. All pumps have a number of critical speeds of which usually only the first two are important. Critical speeds vary inversely with shaft overhanging lengths so that, in general, the longer the shaft, the slower the allowable operating speed and vice-versa. In horizontal pumps shaft overhangs are short and operating speeds are reasonably low so that only the first critical speed needs occasionally to be checked. Vertical pumps, on the other hand, often run between first and second critical speeds and checks are very important and should always be carried out. Most sump pumps operate to depths of 1 to 3 metres. Vertical pumps are hydraulically virtually the same as horizontal pumps so they will not be mentioned any more.

Positive displacement pumps are almost invariably of the piston and diaphragm or piston and cylinder design with inlet and outlet poppet valves. They are employed in pumping through very long pipelines, say from 2 to 50 km because of their ability to generate high pressures, which are well in excess of multi-staged centrifugal pumps. Their design flow rate range is limited from 50 to 1000 litres per second, due mainly to their large physical sizes, both at low and high flows. They are most suitable for transporting slurries with high concentrations of fine particles with a maximum size of about 6 mm. The maximum particle size is dictated by the poppet valves, which can jam in semi-open position by large particles. Poppet valves are high wear items and they need frequent attention and replacement.

In the following we concentrate on horizontal centrifugal pumps only.

CLEAR LIQUID PUMPS
Many textbooks have been written about the design of efficient water, or clear liquid, centrifugal pumps. Their impeller vanes, or blades, are thin so as to offer the minimum obstruction to the flow. The number of vanes can vary from five to ten, depending on pump size, so as to form narrow passageways to correctly guide the liquid from the eye of the impeller to the surrounding volute and impart to it kinetic energy. In these pumps, the casings are mostly of true volute design or of diffuser design, which are efficient converters of kinetic energy to potential energy, i.e. velocity to pressure at the discharge flange. In diffuser pumps there is a ring – the diffuser, interposed between the impeller and the volute – with a multitude of short guide vanes, which even out pressure differences around the casing.

Impellers can be of closed design (with two shrouds) or open design (with one shroud). For critical duties, in order to reduce friction and to achieve the best possible pump performance, impeller surfaces are often machined all over or as much as physically possible and polished. All this attention to detail is justified because a pump will handle a liquid virtually for ever, provided that the pump does not cavitate and that it is made of a material that resists corrosion from the liquid itself.
SLURRY PUMPS

Right from the start we must state that a slurry pump can never be as efficient as a water pump even when they both handle water. Water pump and slurry pump design is based on science but with slurry pumps, science is mixed also with a good sprinkling of art and a designer’s know-how. A good deal of compromise is always necessary between what is desirable and what is possible in terms of wear life and all-up costs. The final textbook for optimum slurry pump design is yet to be written and even powerful computer programs are just beginning to approach this goal. Personal knowledge and past experience is much more important in the design of slurry pump than of water pumps.

Let us consider the impeller once again. Its vanes must be thicker in slurry pumps than in water pumps to allow for wear. Because of this extra thickness, there must be fewer vanes, otherwise the passageways would be too narrow and would affect pump performance. The passageways must be made wide enough to let the largest planned solid particles to pass through without blockage. As a consequence, fluid in this impeller cannot be guided as closely as in a water impeller and this in turn results in reduced pumping head and efficiency. Unlike water pump impellers, slurry hard metal impellers are seldom machined outside and, like rubber impellers, never internally. This extra work would be a waste of money because impellers and liners have a finite life – even though made of hard alloys or resilient elastomers – and must be replaced when pump performance falls off.

Another factor affecting both head and efficiency in slurry pumps is the actual presence of solids in the slurry. Water flowing through a pipe changes velocity – and velocity head – when pipe diameters change. When velocity head increases (or decreases), it proportionally decreases (or increases) the static head – in accordance with Bernoulli’s theorem. This exchange between the two types of energy, kinetic and potential, would go on forever if it were not for friction, which gradually destroys all energy.

When we have solid particles in a flowing slurry, water flows faster than the solids because the solids are moved along only when drag forces, generated by the faster water, overcome gravity forces. The difference between water velocity and solids velocity is called slippage. [This is mentioned also in Section 7, page 1]. The average velocity of the slurry is somewhere between the velocity of water and that of the solids.

Let us imagine a solid particle of a certain volume and an equal volume of water, side by side being accelerated through the impeller – at least for a few moments – each gaining kinetic energy. When they leave the impeller and enter the volute, the liquid slows down and converts its velocity to pressure. The solid particle on the other hand continues travelling until it hits the wall of the volute, or other particles, and expend its energy in collisions, friction and erosion. Of the total slurry, only the water part generates discharge head in the pump, the solids are not contributing anything. [See book reference at the end of this Section]. There is therefore a loss of head per unit mass flow and an extra expenditure of power when pumping solids in comparison to pumping water alone. The magnitude of this loss depends on the actual amount of water displaced by the solids, i.e. concentration, sizing and specific gravity of the solids in the slurry as well as on the size of the pump.

The other important part of the slurry pump is its casing, which takes all the pressure loads. There a two main types of casings: those with internal replaceable liners and those without. The latter may look like those of water pumps although slurry pump casings would always have larger distances between the impeller and the cutwater tongue to ensure free passage of large solid particles in the slurry and to reduce wear. For this reason there is always a considerable amount of recirculation inside a slurry pump casing under a variety of operating conditions. Unlined metal pump casings, or shells, are usually made of hard alloys to resist erosion, or of wear resistant steel so that they can have their worn areas repaired by welding.

The liners can be made of virtually any material which resists erosion. Rubber – or more generally elastomer – impellers and liners are used in pumping small solids up to 10 or 12 mm. Larger particles tend to damage impellers and liners by impact and cutting action. For such particles the impellers and lines are mostly made of hard alloys and polyurethanes.

An important feature of slurry pumps is the provision for some simple means of axial adjustment of the gap between the impeller and the adjoining throatbush seal face. This can be achieved either by axial movement of the shaft, bearing assembly and impeller in the pump or by axial adjustment of the throatbush. Either method assists in maintaining pump performance as the inner component parts wear.
PUMP SEALING
Associated with the impeller-throatbush seal face adjustment is the means of sealing the pump against leakage along the shaft. Most pumps still use the proven and versatile water-fed gland seal, which still provides the lowest sealing costs even though it requires reasonably frequent attention by the operators. When water is scarce, a centrifugal seal is used instead. In many modern pumps handling low concentrations of fine particles, like the limestone slurries in Flue Gas Desulphurisation (FGD) circuits, the mechanical seal is used almost exclusively nowadays.

Most slurry pumps (and many clear liquid pumps) are designed with suction diameters larger than discharge diameters. The simple aim of this design modification is to reduce suction pipe velocity and consequent friction losses, which in turn increase available NPSH. This is particularly important in slurry pumping where the density of the slurry reduces the available NPSH (see: Section 8 – NPSH Considerations).

PUMP SELECTION
Correct pump selection is very important. Slurry pumps are designed to suit specific pumping conditions. Pumps used in the cement and similar industries handle mostly fine particles and at low pressures, so the casings – one piece or split – can be of light construction. In rock pumping the pump casing and impeller must not only perform their hydraulic functions but they must also resist hard knocks, so they must be built thick and strong. Stiffening ribs are often cast integral with the casings to give the pump the necessary strength and keeping the overall weight down. Pumps used in multi-stage operations such as on long tailings disposal pipelines, are built to withstand high pressures rather then shocks.

In pumps with replaceable liners all the strength is provided by the two halves of the split casing when bolted together. The liners are usually of reasonably light construction because they are held in position by the housing, which takes care of all internal and external loads.

Fig.9.1 shows which particle sizes are handled by the basic types of solids handling pumps: slurry pumps, dredge pumps and gravel pumps. As far as particle size is concerned, there are only two major restrictions: (1) elastomer impellers and liners cannot handle particles larger than 10 to 12 mm, particularly if they are sharp-edged and (2) impeller and casing passageways as well as cutwater clearance must be large enough to pass the largest solids dictated by the process.

The rapid damage done to elastomer pump parts by large particles is due to cuts, which they produce on straight impingement surfaces rather than surface erosion. Slurry pumps with metal impellers and liners can usually take particles up to 80 to 100 mm. For still larger particles we turn to gravel or dredge pumps, which have fewer vanes and larger passageways – and unfortunately lower efficiencies – than slurry pumps.

Slurry pumps with powers up to about 300 kW are almost universally driven by fixed speed motors through sets of belts and pulleys to produce different pump speeds and generate various heads. Two reasons for selecting belt drives are their low cost and the ease of changing pump speeds. Another reason is the fact that slurry pump impellers are either cast from very hard alloys, or moulded from soft elastomers. The latter must always have some internal metal skeleton, or reinforcing, for strength. Hard alloy impellers can be machined, but with difficulty and only with special tools on heavy lathes, which are not readily available to most end users. Elastomer impellers cannot generally be machined at all, simply because by reducing the outer diameter, the
elastomer cover is either reduced to an insufficient thickness or, worse still, the cover is removed altogether and the metal reinforcing is exposed to erosion and/or corrosion.

In contrast, impellers for clear liquid pumps are mostly made of easily machinable metals or hard plastics and their outer diameters can be turned down on ordinary lathes – either by the manufacturer or by the end user – to produce different heads with pumps direct coupled to motors at fixed speeds.

A small but growing percentage of slurry pumps above 200 kW are also direct coupled to variable speed motors or to fixed speed motors via variable speed hydraulic couplings or variable frequency controllers. These arrangements are used to compensate for wear and where flow conditions fluctuate and pump speed must be controlled either manually or automatically to make up for these changes and maintain steady flows.

**PUMP SPEED AND WEAR**

Whenever we are dealing with centrifugal pumps we are of necessity involved with flow rates, total heads, consumed power and pump speeds. The first three parameters are closely related to the forth, speed. To make the explanation simple, take a point on the best efficiency line of a pump – such as point 1 of a 6/4 pump in Fig.9.2 – where we have $Q_1$, $H_1$, $\eta_1$ and $N_1$. Let us assume that the system resistance curve coincides with the pump bep line and let us double pump speed to $N_2$, to get point 2 on the bep line. This yields a corresponding new set of values: $Q_2$, $H_2$ and $\eta_2$. If we pump a fluid with $SG=1$ the calculated absorbed powers are $P_{i1}$ and $P_{i2}$ respectively. A different value of $SG$ would change the values of $P_{i2}$ and $P_{i1}$ but not their ratio. All these values are listed in Table 9.1 below. Efficiencies $\eta_1=\eta_2=70\%$.

**Table 9.1 – Pump performance at two speeds.**

<table>
<thead>
<tr>
<th>Point</th>
<th>N (r/min)</th>
<th>Q (L/s)</th>
<th>H (m)</th>
<th>$P_i$ (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>850</td>
<td>56</td>
<td>14.2</td>
<td>14.5</td>
</tr>
<tr>
<td>2</td>
<td>1700</td>
<td>112</td>
<td>56.8</td>
<td>116</td>
</tr>
</tbody>
</table>

Ratio (2/1) 2 2 4 = $2^2$ 8 = $2^3$

As can be readily seen from the tabulation, the values at point 2 can be calculated from the values at point 1 and from the speed ratios by means of the following well known basic affinity laws:

$$Q_2=Q_1(N_2/N_1) \quad \text{or} \quad N_2=N_1(Q_2/Q_1) \quad (9.1)$$

$$H_2=H_1(N_2/N_1)^2 \quad \text{or} \quad N_2=N_1\sqrt[3]{H_2/H_1} \quad (9.2)$$

$$P_{i2}=P_{i1}(N_2/N_1)^3 \quad \text{or} \quad N_2=N_1\sqrt[3]{P_{i2}/P_{i1}} \quad (9.3)$$

Note that the same relationships are valid for speed reductions. The simple point we are trying to make here is that flow rate through the pump is directly proportional to pump speed, head to speed squared and power to speed cubed. Generally speaking if we double the pump speed, we double the flow rate through the pump. At the same time we increase the generated head four times and power consumption eight times. This large power increment is transferred from the impeller to the slurry i.e. to the water and the solids by the impeller vanes. It is of course the solids that cause internal pump wear and it is to be expected therefore that such eight-fold power increment would produce a proportionate increase in wear. This is in fact the case: though internal pump wear can vary considerably between overhauls – even in one pump on the same duty – wear at two speeds is approximately proportional to the ratio of the speeds raised to the power of 3, or a little less. The actual value depends on the relative hardness of the pump wearing parts and the solids pumped.

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PUMPS IN SERIES OPERATION

The above reasoning opens another possibility. Often a slurry pumping system requires a pump to be run at near its maximum design speed. In such a case component cost due to wear can be considerable and it is then often worth investigating the use of two identical pumps in series to replace the single one. Consider again the 6/4 pump, this time operating at point 3 in Fig.9.2 and replacing it with two identical pumps in series, each operating at point 4 i.e. at the same flow rate but at half of the required total head. The combined performance of the two pumps in series is represented by the dashed curve at the top. Assuming SG=1 again, pump performance values at points 3 and 4 are as shown in Table 9.2.

<table>
<thead>
<tr>
<th>Point</th>
<th>N (r/min)</th>
<th>Q (L/s)</th>
<th>H (m)</th>
<th>η (%)</th>
<th>Pi (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1700</td>
<td>100</td>
<td>60</td>
<td>69</td>
<td>111</td>
</tr>
<tr>
<td>4</td>
<td>1300</td>
<td>100</td>
<td>30</td>
<td>69</td>
<td>47</td>
</tr>
<tr>
<td>Ratio (4/3)</td>
<td>0.76</td>
<td>1</td>
<td>0.5</td>
<td>–</td>
<td>0.42</td>
</tr>
</tbody>
</table>

The tabulation shows that two pumps would run at 76% of the speed of the single pump and that each would consume 42% of the power of the single pump. Combined power consumption of the two pumps would therefore be 42x2=84% of that of the single pump, i.e. a power-cost saving of 16% over equal periods. The wearing components of each pump would last 1/0.42=238% of those of the single pump. However, for the two pumps, the combined life of the components must be halved to 119% of those of the single pump, which represents a spare-parts cost saving of 19% over equal periods. There would also be further savings due to less production loss as a result of reduced stoppage time and manpower for replacement of wearing parts.

Another considerable advantage of the reduced speed would be the possibility of using elastomer throatsbushes and impellers instead of hard metal parts. Also note, in Fig.9.2, that the combined two-pump performance curve intersects the system resistance curve at a steeper angle than the single-pump curve. A steeper intersection angle generally locks the duty point more firmly in position.

All these savings and advantages would of course have to be weighed up against the capital cost of the two pumps, drives, motors, starters and inter-stage pipework although each of the two motors would be smaller and cheaper than the single large one. The use of two pumps in series usually has most economic merit when pumping is planned to last a long time.
The following is an extract from page 174 of the book:

"PUMPS AND BLOWERS • TWO-PHASE FLOW"

by A.J. Stepanoff

© John Wiley & Sons, Inc. 1965

**Quote**

**PUMPING SOLID-LIQUID MIXTURES WITH CENTRIFUGAL PUMPS**

**Theoretical Considerations.** It is important to realize that solids suspended in a liquid cannot absorb, store, or transmit pressure energy which is a property of fluids. Pressure exerted by a fluid on the walls of a container is caused by the bombardment of molecules freely moving in a confined space. Molecules of a solid being restricted in their movement by intermolecular cohesion cannot participate in maintaining or transmitting pressure energy, nor can they increase their own kinetic energy (disregarding the energy of elastic compression) when surrounded with liquid carrying the pressure energy. A simple example will illustrate the point. In a standpipe, a pound of solids at the top of the standpipe has as much potential energy of position as a pound of water (referred to the bottom of the standpipe). At the bottom of the standpipe, a pound of water possesses as much potential energy as a pound of water at the top of the standpipe, but a pound of solids at the bottom of the standpipe does not possess any energy referred to the same datum.

If a mixture of solids and water is pumped into the standpipe, a pound of solids would not carry energy into the standpipe except a small amount of kinetic energy which is eventually wasted. In a standpipe filled with a solid-liquid mixture of an average specific gravity of say 1.2, the pressure gage at the bottom of the standpipe will register a pressure 1.2 times that of the clear water. But this pressure increase is due to the extra pressure energy of the liquid required to move and support solids in suspension. This extra pressure energy is derived by the liquid from the impeller, the net effect is as if the impeller was handling a fluid of 1.2 specific gravity. The potential energy of solids is derived from the liquid. An impeller cannot pump solids.

In a steady-flow process solids cannot transmit their kinetic energy to the water since the water particles in every case move with a higher velocity. At the final destination point the kinetic energy of the solids is wasted as well as that of water. The loss is unavoidable and insignificant.

In a horizontal pipe carrying a solid-liquid mixture, solids do not acquire any potential energy but only cause hydraulic losses due to obstruction of the flow of water. This energy loss is increased with volumetric concentration and grain size as is well established in pipe experiments.

**Unquote**
SECTION 10 – Head Ratios and Efficiency Ratios

From the reasoning in Section 9, it follows that we must somehow determine the loss of head and power in a pump handling a water-solids mixture. All centrifugal pumps – of water or slurry design – are routinely works tested in test loops only on water. The simple reason for this is that water is universally available and needs no preparation or conditioning. Also, tests can be reproduced on the same pump anywhere in the world for confirmation of works tests.

It would be nice to be able to test pumps routinely in test loops also on all kinds of slurries. This is usually a costly operation and is therefore carried out only when the job is large enough to warrant it. Preparation and transport of large quantities of dry solids over large distances is expensive since solids must be sieved and mixed in the right proportions. Particle size distribution in the test loop must be checked and adjusted continuously because particles degrade in size due to recirculation and internal attrition.

The only practical way of testing a pump on slurry is at the actual installation where new solids are fed continuously through the pump.

Whichever method is used, the measurements required during a slurry test are basically the same as with water, namely tests at various pump speeds and flow rates. We must first test the pump on water by measuring pump speed N, flow rate Q, suction head Hs, discharge head Hd and power input Pi. We then calculate total heads H and efficiencies, correct them for any speed variations and plot the results on a pump performance graph as shown typically in Fig.10.2 on the next page. All heads are expressed in metres of water.

Next we repeat the test with the pump handling a slurry. We take the same measurements as with water but we also take continuous slurry samples to measure solids concentration Cw, check particle size distribution and make adjustments as required. This time we express the heads in metres of slurry and plot the new results on the same graphs as for water, using the same scales. This is also shown in Fig.10.2.

Head and efficiency values at constant speed and flow rate for slurries are always lower than water values and this is due to the slippage as explained above. As a consequence, if at any fixed speed and flow rate we take the value of the head when pumping slurry and divide it by the head when pumping water we always get a number, which is less that unity. This quotient or ratio is called the Head Ratio HR of the pump for the particular pumping conditions. The same applies to efficiency values for slurry and water. Their quotient is called the Efficiency Ratio ER for the same conditions. Values of Efficiency ratios are either equal to or smaller than Head ratios. The reason for this is that Efficiency ratios reflect the effects of head losses as well as power losses.

Head and Efficiency ratios vary from pump to pump and from slurry to slurry and they can only be obtained from actual tests. Large numbers of HR and ER have been collected over the years by various researchers and, based on these collections, various methods have been devised to predict their values for any planned pumping application. Here we recommend the use of the simple method shown in the Head and Efficiency Ratios Diagram A1–7 in Appendix A1. This should not preclude the use of other proven methods, if preferred.

In the absence of specific test results, Head and Efficiency ratios may be considered constant at most pump speeds and from flows just beyond the pump’s best efficiency point (bep) down to flows about 25 to 30% of bep for most solid-liquid mixtures. If tests are possible, they should be carried out at all flow rates and speeds that might be required at a particular pumping installation.

It should be noted however that Bingham fluids affect the HR in a very different and unexpected way. At any given pump speed the HR has a certain value beyond the bep flow. It maintains the same value as we move towards the left, past the bep and – in rough terms – near half-bep flow the HR starts to drop in value. It then dips down to its lowest value around 25% of bep flow and then it rises again as it approaches zero flow.
It is worth mentioning one such occurrence. In the cement plant mentioned in Section 5 they tried to pump a dense Bingham slurry through a pipe of D=0.15m with a 6/4 pump. The required flow rate was at less than half-bep flow for this pump but they had selected an oversize pump because they expected an increase in production at a later date, which is not unusual. The system friction curve (same as that in Fig.5.4) was drawn together with the static head and then a pump slurry curve, with an assumed constant HR=0.95, was plotted under the published H-Q water pump performance curve. The pump was started and… nothing happened. They changed a couple of pulleys and pump speeds without success and then somebody suggested to try a smaller pump (4/3), with the duty point closer to the pump’s bep. It worked.

When trying to pump slurry, the 6/4 pump was operating in the “dip”, which means that the pump’s actual Hm-Q curve was below the system resistance curve and so the two never intersected, as shown in Fig.10.1. With the 4/3 pump, the dip was further to the left of the duty point.

Had they done some pump testing, they would have had the correct HR for the 6/4 pump and they would not have tried to use it under those conditions.

EXAMPLE: ES10.1
Obtaining HR and ER by testwork

A Warman 14/12 FF-AH pump was tested on water and the standard performance curves WPA 1412A01 were produced. The pump operated in a Mill Discharge duty feeding a cyclone under the following conditions:

Pump speed:
N = 510 r/min
Duty flow rate:
Q = 500 L/s
Solids:
S = 2.65, d50 = 0.5 mm, dmax = 12 mm
Solids concentrations:
Cw = 60%, Cv = 36%
Impeller diameter:
Di = 965 mm
Particle to impeller ratio:
d50/Di = 0.0005

The pump was tested at the duty point (500 L/s) and at 400 L/s, by partial throttling of the discharge valve at the cyclone inlet.

Measurements were taken of heads, flows and power and the new Head-Flow (Hm) and Efficiency-Flow (ηm) curves plotted. The Head ratio was found to be 0.85 and the Efficiency ratio 0.82. This is all shown in Fig.10.2. These values are very close to the values obtained from the HR and ER Diagram A1-7 in the Appendix A1.
SECTION 11 – Examples of Slurry Pumps and Pipelines

This is where everything gets put together in small bits from various parts of the Manual. In what follows we show a few problems and solutions from actual duties. This is where the reader could add additional pages with his/her own examples of particularly interesting and/or tricky cases of slurry handling.

EXAMPLE: ES11.1
Ball Mill, discharge Pump and Cyclone circuit

At many mines they must crush and grind all the ore to below a certain limiting size before it can be put through an appropriate separation section to recover the final product. A plant metallurgist would normally determine this limiting size, called the “split”, to suit the particular process. When flotation cells are used for this purpose, the split is usually around 100 µm, because the final product can be separated efficiently from the tailings only when particles are below this size.

The grinding section often consist of a Semi-Autogenous (SAG) Mill followed by a vibrating screen and a Ball Mill. From the vibrating screen, the oversize (+10 mm) material is recycled back to the SAG Mill while the –10 mm portion proceeds to a cyclone-feed pump hopper. This hopper also collects the discharge from the Ball Mill itself. The pump collect and delivers all this mixture to a cyclone (or hydro-cyclone) above the Ball Mill, where the particle size split occurs. The cyclone overflow, containing the fines or “product”, progresses to the flotation circuit, whereas the cyclone underflow, containing the coarser material, is fed back to the Ball Mill and the process is repeated with fresh material from the SAG Mill.

This example comes from a copper mine. The specific gravity of copper ore is $S=2.85$, the flow of slurry fed to the cyclone is $Q=61.7$ L/s, the solids concentration in the slurry is $C_w=40\%$ (and $C_v=19\%$) and the specific gravity of the slurry is $S_m=1.35$.

Particle size distributions of the cyclone feed, the cyclone underflow and the cyclone overflow (the product) are shown in Fig. 11.1. Here we are only interested in the particle distribution of the cyclone “feed” stream. From the graph we get $d_{50}=60\mu m$, $d_{50}=250\mu m$ and $d_{50}=750\mu m$. So: $d_{50}/d_{20}=12.5 > 5$.

In order to produce the required split the selection of the correct type and size of cyclone is usually left to cyclone “experts”. The Warman 500CVX CAVEX® cyclone was suggested for this duty. It has the following dimensions: housing dia.=0.500 m, inlet dia.=0.160 m, vortex finder dia.=0.185 m and spigot dia.=0.100 m. At the nominated flow, the cyclone requires an inlet pressure $P_c=65$ kPa.

The pumping circuit is shown in Fig. 11.2 on the next page. The height of the cyclone inlet above the mean water level in the pump sump is $Z=16$ m, the internal diameter of the pipe is $D=0.150$ m and the total equivalent length of pipe is $L=30$ m.
The pump selected for the duty is a 6/4 E-AH Warman slurry pump with a hard-metal impeller with outer diameter $D_i=0.365$ m. Its performance curves WPA64A01 for water and slurry together with the system resistance curves are shown in the abridged graph in Fig.11.3.

A flow of $Q=61.7$ L/s in a steel pipe of $D=0.150$ m, yields an actual pipeline velocity of $V=3.8$ m/s. From Fig.11.1 we see that $d_{80}$ is more than 5 times $d_{20}$ and so we use the modified graph from Fig.7.2 to obtain Durand’s pipe velocity limiting factor $F_L=1.1$ for solids $S=2.85$ with particle sizing $d_{50}=250\mu m$ and concentration $C_v=19\%$. The limiting settling velocity, from equation (7.1), is then $V_s=2.3$ m/s, which is smaller than $V$ and so there will be no settling in the pipe.

Next we must obtain the system resistance curve for the pipeline. Using Diagram A1.1 from Appendix A1 with a relative pipe wall roughness $e/D=2.8\times10^{-4}$ the Darcy friction factor is $f=0.016$ and the friction head loss $H_f=1.98$ m slurry. The cyclone pressure $P_c=65$ kPa and the corresponding cyclone head, by Equation (3.4) is:

$$H_c=\frac{65000}{(1350\times9.81)}=4.91\text{ m slurry.}$$

The static head is $Z=16$ m so the Total Head required from the pump is:

$$H_m = Z + H_f + H_c = 16.00 + 1.98 + 4.91 = 22.9\text{ m slurry.}$$

Next we find the Head and Efficiency Ratios for the pump and we use Diagram A1.7 from Appendix A1 to estimate these values. The particle-size/impeller-diameter ratio is $d_{50}/D_i=0.0007$. Therefore the estimated head and efficiency ratios are:

$$HR = 0.88 \quad \text{and} \quad ER = 0.88.$$

Fig.11.3 shows a portion of the performance curves WPA64A01 for the selected pump when pumping water. System resistance and cyclone head when pumping slurry are also shown.

The pump Total Head when pumping water at the required $Q$ is:

$$H_w = \frac{H_m}{HR} = \frac{22.9}{0.88} = 26.0\text{ m water.}$$

The pump speed required to handle 61.7 L/s of water against this head, from Fig.11.3, is 1400 r/min.

The pump efficiency when pumping water is $\eta_w=69\%$ and when pumping slurry it is $\eta_m=69\times0.88=60.7\%$.

The power input required by the pump is given by:

$$Pi = Q.H_m.S_m/\eta_m = 61.7\times22.9\times1.35/(1.02\times60.7) = 30.8\text{ kW.}$$

A 37 kW motor would give us a 20% contingency margin.
EXAMPLE: ES11.2
Dredging for valuable Minerals and disposing of Tailings

Many valuable minerals – like rutile or ilmenite (both ores of titanium), tantalum and zirconium – are found in large deposits of fine sands along ocean beaches in many parts of the world. Beach mining is a fairly simple operation when compared to rock mining, as it requires no heavy dumper trucks, nor any crushing or grinding machinery. The main equipment usually consists of (1) a suction dredge – sometimes with a motor-driven cutter to cut root of trees – (2) a floating concentration plant with a number of pumps and banks of spirals to separate the various minerals and (3) some means of disposing of the tailings. Often this is done by jet stacking.

Figure 11.4 shows a schematic of such a beach-mining operation. A suitable artificial pond is first excavated in the sand, parallel to the sea-shore and at a safe distance from it, deep enough under the water table so that it fills automatically with ground water. A dredge, a floating plant and a floating pipeline – to connect the two – are then launched in the pond. The dredge pump sucks up the sand and water from the bottom of the pond as the dredge is pulled across the dredging face, by means of winches and cables. When the whole dredging face has been traversed, the dredge is pulled forward and the operation then continues in the opposite direction across the face. This essentially lengthens the pond. The sand slurry is pumped to the concentrating plant through the floating pipeline to a rotary or vibrating screen, where occasional roots and rocks are removed from the sand, which is collected in an open hopper. This sand is then pumped to various parts of the separation process – not considered in detail here – where the useful minerals are removed. The barren sand tailings are then collected in another hopper and then pumped and jetted overboard through one or more inclined pipes and nozzles from the rear end of the floating plant. The plant – like the dredge – is moved back and forth across the pond by means of winches and cables to distribute the tailings evenly across the width of the pond. In this way the pond remains of the same width and length as it progresses (together with all the floating equipment) through continuously new virgin ground. Trees and bushes are finally re-planted on the deposited tailings to return the open sand surface as quickly as possible to its original, or better, environmental state.

DREDGING

A customer requires to process a minimum of 150t/h of sand and is considering purchasing a dredge with a Warman 10/8 F-G Gravel pump driven through a gearbox by means of a 250 kW diesel engine. The pump suction pipe consists of 7m of steel pipe and 3m of reinforced rubber hose, both of 0.250m diameter. The pipe is pivot-mounted and can be tilted from above water line (for maintenance purposes) to 30° below horizontal, at which its mouth is 3m vertically below the water surface. The discharge pipe on the dredge is 0.250m diameter and 10 m long with a 0.200x0.250m divergent piece and 3 long bends.

The customer needs to dredge and pump sand to a floating plant as shown above and needs to check the dredging capacity of the unit and the maximum distance pumped. The dredged sand is reasonably free-flowing so that a cutter head is not required at the mouth of the suction pipe. The maximum solids concentration is expected to be approximately Cw=30%.

Data:

**Ambient:** Hatm=10.3m water

**Water:** Sw=1, Tw=25°C maximum, Hvap=0.3m

**Sand:** S=2.65, d50=200µm, Cw=30%, Cv=14%, Sm=1.24.
Table 11.1 shows the conditions on the suction side of the pump at various flow rates $Q$, when pumping water and slurry. Pipe internal diameter is $D_s=0.250\text{m}$ and total equivalent length $L_s=15\text{m}$.

### Table 11.1 – Suction Heads and $NPSHa$

<table>
<thead>
<tr>
<th>$Q$ [L/s]</th>
<th>discharge</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_s$ [m/s]</td>
<td>pipe velocity</td>
<td>2.0</td>
<td>3.1</td>
<td>4.1</td>
<td>5.1</td>
</tr>
<tr>
<td>$H_{vs}$ [m]</td>
<td>velocity head</td>
<td>0.2</td>
<td>0.5</td>
<td>0.9</td>
<td>1.3</td>
</tr>
<tr>
<td>$H_i = H_{vs}$ [m]</td>
<td>head loss at suction pipe inlet</td>
<td>0.2</td>
<td>0.5</td>
<td>0.9</td>
<td>1.3</td>
</tr>
<tr>
<td>$H_{fs}$ [m]</td>
<td>friction head loss – using Addenda diagram 13.1</td>
<td>0.2</td>
<td>0.4</td>
<td>0.7</td>
<td>1.1</td>
</tr>
<tr>
<td>$Z_s$ [m]</td>
<td>suction static head</td>
<td>$-0.5$</td>
<td>$-0.5$</td>
<td>$-0.5$</td>
<td>$-0.5$</td>
</tr>
<tr>
<td>$H_{s'} = Z_s – H_i – H_{fs}$ [m water]</td>
<td>$-0.9$</td>
<td>$-1.4$</td>
<td>$-2.1$</td>
<td>$-2.9$</td>
<td></td>
</tr>
<tr>
<td>$H_{atm}$ [m water]</td>
<td>atmospheric head</td>
<td>10.3</td>
<td>10.3</td>
<td>10.3</td>
<td>10.3</td>
</tr>
<tr>
<td>$H_{vap}$ [m water]</td>
<td>water vapour head at $T_w=25^\circ\text{C}$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>$H_{PSHa} = (H_{atm} – H_{vap}) – H_{s'}$ [m water]</td>
<td>9.1</td>
<td>8.6</td>
<td>7.9</td>
<td>7.1</td>
<td></td>
</tr>
</tbody>
</table>

... when pumping slurry – as for water, above, except as noted:

| $Z_w$ [m] | depth from water level to suction pipe inlet | 3.0  | 3.0  | 3.0  | 3.0  |
| $Z_{bc} = Zw.Sw / Sm$ [m slurry] | balanced slurry column | 2.4  | 2.4  | 2.4  | 2.4  |
| $Z_{uc} = Zw – Z_{bc}$ [m slurry] | unbalanced slurry column | 0.6  | 0.6  | 0.6  | 0.6  |
| $H_{s} = H_{s'} – Z_{uc}$ [m slurry] | $-1.5$ | $-2.0$ | $-2.7$ | $-3.5$ |
| $NPSHa = (H_{atm} – H_{vap}) / Sm – H_{s}$ [m slurry] | 6.8  | 6.3  | 5.6  | 4.8  |

The calculations are straightforward and self-explanatory. $H_{s'}$ represents the suction head existing at the pump suction flange when pumping water. This consists of the sum of the static head $Z_s$, the head loss at inlet $H_i$ and the friction loss $H_{fs}$. When pumping slurry, the suction head $H_s$ is increased by an additional unbalanced slurry column $Z_{uc}$, which can be explained with the help of Fig.11.5.

The vertical projection of the suction pipe, below the water surface, is equal to $Z_w$. The pipe is surrounded by water of $S_w=1$, while inside the pipe we have slurry of $S_m=1.24$. The U-tube manometer shown, replicates portion of the suction pipe. The column of water $Z_w$ is balanced by a column of slurry $Z_{bc} = Zw.Sw/Sm$.

This means that the pump does not have to lift the slurry column $Z_{bc}$ but it must only lift the unbalanced portion of the column, ie.:

$$Z_{uc} = Zw – Z_{bc} \quad (11.1)$$

Table 11.2 shows the conditions on the discharge side of the pump at the same flow rates. Pipe internal diameter is $D_d=0.250\text{m}$ and total equivalent lengths $L_d=150\text{m} / 400\text{m} (=\text{min} / \text{max})$.

### Table 11.2 – Discharge Heads

| $V_d$ [m/s] | pipe velocity | 2.0  | 3.1  | 4.1  | 5.1  |
| $H_{vd}$ [m] | velocity head | 0.2  | 0.5  | 0.9  | 1.3  |
| $H_{fd}$ [m] | friction head loss – using Addenda diagram 13.1 | 2.6 / 8.7 | 5.9 / 19.7 | 10.5 / 35.0 | 16.3 / 54.5 |
| $H_e = H_{vd}$ [m] | head at pipe exit | 0.2  | 0.5  | 0.9  | 1.3  |
| $Z_d$ [m] | discharge static head | 5.0  | 5.0  | 5.0  | 5.0  |
| $H_d = \pm Z_d + H_{fd} + H_e$ [m water] | 7.8 / 13.9 | 11.4 / 25.2 | 16.4 / 40.9 | 22.6 / 60.8 |

Table 11.3 shows the total Static Heads for water and slurry at the same conditions.
Table 11.3 – Total Static Heads

... when pumping water

\[ Z' = Z_d - Z_s' \text{ [m]} \]

\[ \begin{array}{cccc}
5.5 & 5.5 & 5.5 & 5.5
\end{array} \]

... when pumping slurry

\[ Z = Z' - Z_{uc} \text{ [m]} \]

\[ \begin{array}{cccc}
6.1 & 6.1 & 6.1 & 6.1
\end{array} \]

Finally, Table 11.4 shows the total heads for water.

Table 11.4 – Total Heads

... when pumping water

\[ Q \text{ [L/s]} \] discharge

\[ H = H_d - H_s \text{ [m water]} \]

\[ \begin{array}{cccc}
100 & 150 & 200 & 250 \\
9.3 / 15.4 & 13.4 / 27.2 & 19.1 / 43.6 & 26.1 / 64.3
\end{array} \]

Fig.11.6 is an abridged copy of Warman Performance curves WPA-108A11 for the 10/8 F-G pump whose impeller diameter is \( D_i = 0.533 \text{m} \).

Values from Table 11.4 were used to plot the System Resistance water curves (\( p-b \) and \( p-n \)) and the two discharge pipe lengths. Note that point ‘\( p \)’ is the total static head \( Z \) and not \( Z' \).

The Durand limiting settling velocity \( V_L \) was calculated from Diagram A1.4 from Appendix A1. The corresponding flow rate was \( Q_L = 144 \text{L/s} \), which corresponds to point ‘\( i \)’ on the diagram.

Values from Table 11.4 were used to plot the System Resistance slurry curves (\( e-d-c \) and \( f-g-m \)) were then drawn above these points as explained in Fig.6.2 of Section 6.

Point ‘\( i \)’ (and points ‘\( g \)’ and ‘\( d \)’ above it) determines the minimum flow rate required in the pipeline to prevent solids deposition.

The Total Head at \( Q=220 \text{ L/s} \) is \( H_m=45.0 \text{m slurry} \) from the Addenda Diagram 13.6 and for an impeller diameter \( D_i = 0.533 \text{m} \) the Head Efficiency Ratios are \( HR=ER=0.9 \).

\[ P_i = \frac{Q.H_m.S_m}{(1.02.\eta_m)} = \frac{220 \times 45.0 \times 1.24}{(1.02 \times 54)} = 220 \text{ kW} \]

The 250 kW motor has a 13% margin over the maximum required and is therefore quite satisfactory.
JET STACKING

Having got the sand into the processing plant and extracted any useful mineral ores, the useless sand must then be returned back the beach. Fig.11.7 shows a typical basic tailings jet stacking arrangement. Calculations of various heads are carried out in the normal fashion and they are not repeated here. The main purpose of this Example is to point out some unusual features of such a pumping unit.

The tailings are usually collected in one or two constant-density pump hoppers from where they are pumped overboard through an inclined pipeline and a nozzle behind the floating plant. The jetting pump (or pumps) is smaller than the dredge pump, because the pumping density is usually higher (around Cw=60%) to ensure quick dewatering of the slurry as soon as it hits the ground. Occasionally two pumping units are used in parallel for a more even spreading of the tailings.

A constant-density pump hopper consists of a standard hopper with a standard suction pipe pointing through the wall to the bottom. There is however an important modification in this pipe. It has a vertical tee-piece connection with the added (short) leg pointing vertically up. This leg has a simple knife-gate valve at the top. The sand from the incoming slurry settles at the bottom of the hopper and separates from the water, which remains above it. The valve lets water in and it can be adjusted as required so that the water and the sand mix into a slurry of the desired constant density so that a steady pumping velocity \( V_d \) (above \( V_L \)) can be maintained to prevent settling problems in the discharge pipe.

When the slurry expelled from the nozzle hits the ground, the wet sand settles on a slope with a shallow angle of repose (usually around 8°). The point of impact must be a suitable horizontal distance from the end of the plant so that the sand settles below its stern but does not touch it or impede its sideways movement. This can be all worked out from simple geometry once the height of the jetted deposit above water level is known. The total distance is the sum of two parts: (1) the horizontal projection of the inclined pipe and (2) the throw of the jet \( L_j \).

The maximum jet throw (see Fig.11.7) is obtained when the nozzle is inclined approximately at \( \theta_j = 50° \) above horizontal. It may be calculated from:

\[
L_j = 2.\cos\theta_j(\sin\theta_j/g)^2V_j^3 = 0.00739V_j^3 \text{ [m]}
\]

The exit velocity of the jet from the nozzle is usually around \( V_j = 15 \text{ m/s} \) in order to keep nozzle wear life and its cost reasonable. The nozzle diameter can be calculated from the required flow rate \( Q \) and the jet velocity \( V_j \) and the Velocity Head \( V_j^2/(2g) \) must be added to the Static and Friction Heads to get the Total Head of the System.
APPENDIX 1 – Diagrams for Use in Slurry Pumping

The following pages contain some useful Diagrams, which can be used as short-cuts for obtaining Friction losses in pipelines, Properties of slurries, Limiting settling velocities in pipelines and Head ratios and Efficiency ratios for pumps when pumping slurries.

The graphs are:

A1–2  Darcy Pipe Friction Diagram
A1–3  Hazen-Williams Pipe Friction Diagram
A1–4  Properties of Mixtures of Solids and any Liquid
A1–5  Durand’s Limiting Settling Velocities
A1–6  Wilson’s Deposition Velocities
A1–7  Head Ratios and Efficiency Ratios for pumping Solids
EXAMPLES:

Pumping at $V=3 \text{ m/s}$ through steel pipe $D=0.25 \text{ m}$ with wall roughness $e=4\times10^{-5} \text{ m}$.

1. Water at $T=20^\circ\text{C}$:
   - Kinematic viscosity: $v=10^{-6} \text{ m}^2/\text{s}$ ($=1 \text{ cSt}$).
   - Start with $D$ from bottom left and bottom right and follow the arrows to the end to get $f=0.0145$ and:
     $$H_f=3.0 \text{ m water }/ 100 \text{ m pipe}.$$  

2. Linseed oil at $T=70^\circ\text{C}$:
   - Kinematic viscosity: $v=10^{-5} \text{ m}^2/\text{s}$ ($=10 \text{ cSt}$).
   - Start as above but stop at $D$, read $N=7.8\times10^5$, divide it by $10$ (cSt) to get new $N=7.8\times10^4$, enter it at $c$ and continue to the end to get $f=0.0197$ and:
     $$H_f=3.7 \text{ m linseed oil }/ 100 \text{ m pipe}.$$  

(Note: cSt=centistokes)

**Darcy Pipe Friction Diagram**

Pipe velocity $V$ is given by:

$$H_f = f \frac{L V^2}{D 2g} \text{ [m fluid]}$$

where:

- $f =$ Friction factor $[-]$
- $L =$ Pipe length $[\text{m}]$
- $V =$ Pipe velocity $[\text{m/s}]$
- $D =$ Pipe ID $[\text{m}]$
- $g = 9.81 \text{ m/s}^2$
DURAND’S LIMITING SETTLING VELOCITY DIAGRAM

EXAMPLE:
- \( d_{50} = 0.5 \) mm
- \( C_v = 5\% \)
- \( D = 0.2 \) m
- \( S = 2.65 \)

\[ V_L = \frac{F_L \cdot (2gD \cdot S - 1)}{2} \]

\( V_L = 3.4 \) m/s
\( (F_L = 1.34) \)
WILSON’S DEPOSITION VELOCITIES

EXAMPLE:
Solids of specific gravity $S = 2.65$ and particle size $d_{50} = 0.19$ mm are to be pumped through a pipe of diameter $D = 0.150$ m.
The highest expected deposition velocity is:
$$V_c = 1.95 \pm 0.40 \text{ m/s}.$$
HEAD RATIOS AND EFFICIENCY RATIOS FOR PUMPING SOLIDS

SOLIDS MEDIAN PARTICLE SIZE: \( D_{50} \) [mm]

EXAMPLE:
- \( d_{50} = 0.5 \) mm
- \( D = 400 \) mm
- \( d_{50}/D = 0.00125 \)
- \( S = 2.65 \)
- \( C_v = 30\% \)
- \( H_R = 0.79 \)
- \( E_R = 0.75 \)

SOLIDS CONCENTRATION IN SLURRY
BY TRUE VOLUME: \( C_v \) [%]

PARTICLE/IPPELLER RATIO: \( d_{50}/D \) [-]

PUMP IMPELLER DIAMETER: \( D_i \) [mm]

EFFICIENCY RATIO: \( E_R \) [-]

SPECIFIC GRAVITY: \( S \) [-]
APPENDIX 2 – References Publications

The publications listed below represent only a small selection of books dealing with slurry pumping. We encourage the reader to seek other publications to keep abreast of the latest developments.


The Transportation of Solids in Steel Pipelines – Colorado School of Mines Research Foundation, 1963


Technical Bulletins – Warman International Ltd., 1990s

APPENDIX 3 – Photographs of Pump Installations

1

WARMAN 10/8 T-AHPP gland sealed pumps.
Two trains of 7 stages. Tailings application.
Rubber lined casings and high efficiency, hard-metal impellers.
Duty: Q=930 m³/h, Hm=55 m per stage, Cw=55%.
Drives: 315 kW, 740 r/min motors, direct coupled.
ASH PUMP 22x20 SRH pumps.
Two trains of 3 stages (one train shown). Tailings application.
Rubber lined casings and impellers.
Duty: Q=3600–5000 m³/h, Hm=24–31 m per stage, Sm=1.2–1.3.
Drives: 750 kW motors with gear reducers and Variable Frequency Speed Controllers.
WARMAN 20/18 TU-AH pumps.
One pump per processing circuit. Mill discharge application.
Rubber lined casings, hard-metal impellers and throat bushes.
Duty: \( Q = 3200 \text{ m}^3/\text{h} \), \( H_m = 32.5 \text{ m} \), \( C_w = 62\% \).
Drives: 600 kW motors and gear reducers.
GALIGHER 5000 vertical 3.5 SRA 2300x72" pumps.

One operating, one on stand-by. Salt brine pumping.

High temperature EPDM liners and impeller

Duty: Q=91 m³/h, Hm=22.9 m, Sm=1.2.

Drives: 20 kW, 1800 r/min motors and V-belts.
WARMAN 550 U-SHD pumps with high pressure gland seals.
Two sets of 2 stages. Oilsands application
Hard-metal lined casings and impellers.
Duty: Q=5680 m³/h, 54.8 m per stage, Sm=1.65, handling particles up to 100 mm.
Drives: 1800 kW motors and gear reducers.
ASH PUMP 6x6 SHR pump.
Thickener underflow. Sand and gravel operation.
Rubber lined casing, hard-metal impeller.
Duty: Q=190 m³/h, Hm=28 m, Sm=1.35.
Drive: 37 kW, 1800 r/min motor and V-belts.
7

**WARMAN 800 TY-GSL mechanically sealed pumps.**

One pump per bank of sprays in Recycle vessel. Flue Gas Desulphurisation (FGD) application.
Rubber lined casings and hard-metal impellers and throat bushes.

Duty: 11,520 m³/h, Hm=23.2 m, Cw=25%.

Drives: 1150 kW motors and gear reducers.
ASH PUMP 350 MCH pump.
Ball Mill discharge application.
Rubber lined casing, hard-metal impeller.
Duty Q=1590 m³/h, Hm=14 m, Cw=75%.
Drive: 225 kW, 1200 r/min motor and V-belts.
WARMAN 550 U-SHDU gland sealed pump.
Mill discharge application.
Hard-metal pump casing and impeller; unlined.
Duty: Q=5220 m³/h, 37 m, Cw=72%
Drive: 1200 kW motor with Variable Frequency Speed Controller, direct coupled.
ASH PUMP 250 MCH Low Flow pump.
Wet Crusher discharge application.
Special cut-resistant rubber lined casing, hard-metal impeller and throat bush.
Duty: $Q=360\,\text{m}^3/\text{h}$, $H_m=18\,\text{m}$, $C_w=36\%$, coarse grind.
Drive: 90 kW motor, V-belt drive.
APPENDIX 4 – Slurry Pumping Glossary

Here is a mini-dictionary of terms and definitions used in Slurry Pumping and in Minerals Process Industries.

**Affinity Laws** (also **Affinity Rules**) – Mathematical relationships of pump speed to flow rate, developed head and input power. When speed changes from \( N_1 \) to \( N_2 \), flow rate changes from \( Q_1 \) to \( Q_2 = Q_1 \left( \frac{N_2}{N_1} \right) \), head from \( H_1 \) to \( H_2 = H_1 \left( \frac{N_2}{N_1} \right)^2 \) and power from \( P_1 \) to \( P_2 = P_1 \left( \frac{N_2}{N_1} \right)^3 \).

**Beneficiation** – The process of refining ores and raw materials.

**Carrying Medium, in a slurry** – Any homogenous fluid used to suspend solids during flow.

**Characteristic Curve** – A curve showing the relationship between pump heads and flow rates at a given speed.

**Comminution** – The reduction of size of solids particles by means of mill grinding, attrition or any other size reduction process.

**Concentrates** – The valuable portions of ores and raw materials obtained by a beneficitation process.

**Concentration by Weight** (or **Mass**) – The weight (mass) of solids present in a two-phase mixture taken as a percentage of the weight (mass) of the total mixture.

**Concentration by Volume** – The volume of solids present in a two-phase mixture taken as a percentage of the volume of the total mixture.

**Consistency** – A measure of the resistance of a slurry to shear. As with non-Newtonian fluids, the dynamic viscosity (being the ratio between shearing stress and rate of shearing strain) is not constant and must be determined for any particular flow rate.

**Density of Material** – Mass of material in kilograms per cubic metre. Material may refer to fluid, solid or mixture.

**Drag Coefficient** – A measure of the forces resisting the movement of a solid particle in a fluid.

**Efficiency** – The ratio of energy transferred to a pumped fluid and energy supplied to the pump shaft.

**Fines** – Small particles of ore or metal.

**Flocculation** – The use of high molecular weight synthetic organic polymers which are added to a suspension of finely divided solids in water, in very small quantities, which increase the settling rate of the suspended material by combining fine particles together to form a larger agglomerate.

**Flotation** – A continuous process in which cells are arranged in series forming a bank. Pulp enters the first cell of the bank and gives up some of its valuable mineral as a froth. Air bubbles overflow from this cell taking the desired mineral with them. The underflow from this cell passes to the second cell where more mineralized froth is removed, continuing down the bank, until barren tailings overflow and the underflow passes to the last cell in the bank. Pulp passing through each cell is drawn upwards into the rotor by the suction created by the rotation. Slurries are thoroughly mixed with the air before being broken into small, firm bubbles by the disperser, which surrounds the rotor, by abruptly diverting the whirling motion of the pulp.
Flow by Saltation – Turbulent flow of a slurry with solids too coarse to be fully suspended by turbulence. The solid particles travel by consecutive bounces and leaps on the bottom of the pipe.

Flow with Stationary Bed – The flow of a slurry in a pipe with restricted area due to a deposit of solid particles on the bottom of the pipe. The deposits often have dunes, both stationary and moving.

Flow rate (also Rate of Flow, Discharge and Capacity) – The volume of fluid a pump delivers during a certain period of time. This can be expressed in a variety of ways, such as litres per second, cubic metres per hour, gallons per minute, etc. Slurry pump applications are often specified in terms of tonnes of solids per hour and in such cases they must be converted to the appropriate volumetric units.

Friction loss – The resistance to flow within a pipe and fittings. It is a function of the rate of flow, viscosity of the liquid, and pipe diameter and is expressed in metres of fluid pumped.

Head – The potential and/or kinetic energy that exists in a fluid. It is usually expressed in metres of fluid pumped.

Heterogeneous Slurry – A flowing slurry in which the concentration of solid particles is not uniform but increases towards the bottom of the pipe.

Homogenous Mixture or Slurry – A two-phase mixture of solid particles and fluid in which the two phases cannot be separated by mechanical means and in which the distribution or concentration of solid particles is uniform across the section of the pipe. In practice this is really a “model” slurry with uniform properties that we assume in order to simplify system calculations.

Hydraulic Gradient – The friction head loss per unit length of pipe.

Laminar Flow – In laminar flow, all the fluid particles proceed along parallel paths and there is no transverse component of velocity. The orderly progression is such that each particle follows exactly the path of the particle proceeding it without any deviation. Three-dimensional laminar flow can be viewed as concentric thin shells along the pipe axis. Laminar flow is associated with low velocities and viscous sluggish fluids.

Limiting settling velocity – The velocity of heterogeneous slurry in a horizontal pipeline, according to Durand’s studies, below which solids of a certain size, density and concentration begin to settle along the bottom.

Liner, Drive side (also Liner, Frame Plate) – A cast hard alloy or moulded rubber component lining the drive side of the pump casing.

Liner, Suction side (also Throatbush) – A cast hard alloy or moulded rubber component lining the suction side of the pump casing.

Liner, Volute (also Liner, Casing Shell) – A cast hard alloy or moulded rubber component lining the volute of the pump casing.

Mesh – The number of openings in a screen per linear mm (metric measurement) or inch (imperial measurement).

Mill – Machinery used for crushing and grinding ores.

Mixture – See Slurry

Mud, Paste or Sludge – A two-phase mixture of solid particles and fluid which is not a true homogenous mixture.

Newton – The force which imparts an acceleration of one metre per second squared to a mass of one kilogram.

Newtonian Fluid – A fluid exhibiting a direct relationship between shearing stress and rate of shear in laminar flow. The flow of Newtonian fluids is completely characterized by a single property, the viscosity, which is the ratio of shear stress and velocity gradient at any flow.
Non-Newtonian Fluid – A fluid exhibiting a variable relationship between shearing stress and either the rate of shearing strain (Bingham Plastic, Pseudoplastic and Dilatant fluids) or the duration of shear, i.e. time dependence (Thixotropic and Rheopectic fluids). The flow of non-Newtonian fluids cannot be characterized by a unique property. The apparent viscosity of a non-Newtonian fluid is the ratio of shear stress and velocity gradient at one particular flow rate. It has no meaning at any other flow rate. Most solutions and suspensions at low concentrations behave as a Newtonian fluids, changing to non-Newtonian behavior when certain critical solids concentrations are reached. In the case of suspensions, critical concentrations depend on particle size and shape and on the degree of dispersion of the particles. For meaningful results, laboratory tests of Non-Newtonian samples should be carried out only with fresh slurry samples from the actual system being evaluated.

NPSH – Net positive suction head. The total suction head in absolute units at the pump suction flange less the absolute vapor pressure of the liquid.

Particle Settling Velocity – The terminal, free-fall velocity of a solid particle falling in calm, clear water. The particle settling velocity is also used to describe the settling of slurries with low concentration of solids.

Reynolds number – A dimensionless number defining a fluid flow, expressed by the ratio of inertia forces and viscous forces.

Screen Analysis – The separation of solid particles into certain size fractions by means of standard sieving screens.

Shaft Sleeve – A tubular hardened metal piece fitted over the shaft to protect it from wear by the gland packing in the stuffing box.

Shear rate – see Velocity gradient.

Slimes – A rock product which contains particles of not more than 5% +300 µm size nor less than 50% –75 µm size, and which includes all fine material produced by a comminution process.

Slurry – Strictly, a two-phase mixture of solid particles and fluid in which the two phases will not chemically react with each other and the two phases can be separated readily by mechanical means. However, in this Manual the terms “slurry” and “mixture” are used more loosely and interchangeably to describe a mix of any solids and liquids.

Slurry Settling Velocity – The velocity at which solids start to separate out of a suspension of solid particles before compaction is achieved.

Solids (also Solid Particles) – Fragments of solid materials which are chemically inert and will not react with the fluid in which they are suspended.

Specific Gravity – The ratio of the weight (or mass) of a substance to the weight (or mass) of an equal volume of water.

Specific Speed – A design index number used to classify pumps by impeller type and proportion. It is defined as the speed in revolutions per minute at which a geometrically similar pump would operate to deliver one unit of flow at one unit of head.

Stuffing Box – A portion of the pump casing through which the shaft extends and which holds a sealing device to prevent leakage. The sealing device can be a soft packed gland or a seal adaptor for a mechanical seal.

System Head curve (or System Resistance curve) – A curve showing the relationship between total heads required by a pumping system and their respective flow rates.

Transition Velocity – The velocity at which the flow of a slurry changes from one flow regime to another, such as from homogenous to heterogeneous flow or from laminar to turbulent flow.

Turbulent Flow – Flow at high Reynolds numbers with fluid and solid particles moving with random motion transverse to the main flow direction.
Velocity – Rate of motion in a given direction expressed in metres per second.

Velocity at limit of stationary deposition – Velocity of a heterogeneous slurry in a horizontal pipeline, according to Wilson’s studies, above which solid particles of a certain size and density will be picked up from the bottom and carried along by the flow.

Velocity gradient (also Shear rate) – A fluid in a pipe under laminar flow conditions moves axially in thin layers or concentric fluid shells. Velocities of successive shells increase from zero at the pipe wall to a maximum at the centre of the pipe. At any given radius ‘r’ there is a shell with velocity ‘v’. At the adjacent radius, reduced by ‘dr’, the respective velocity increase is ‘dv’. The velocity gradient is the ratio dv/dr. Note that the only useful velocity gradients are the ones ‘at the pipe wall’. These are the only values used in viscosity calculations.

Viscosity – Viscosity is the property of a Newtonian fluid in laminar flow to resist the relative displacement of adjoining layers, or concentric shells in a pipe. This resistance is due to internal friction and molecular attraction between particles which, when induced to flow, causes shear stress between adjacent layers.

Viscosity, Apparent – It is the Dynamic viscosity of non-Newtonian fluid, such as homogenous mixtures, muds, pastes, etc. Apparent viscosity changes value at different flow rates.

Viscosity, Dynamic – In Newtonian fluids it is the ratio of shearing stress and velocity gradient expressed in Pascal-seconds. Dynamic viscosity has a constant value at all flow rates.

Viscosity, Kinematic – In Newtonian fluids it is the ratio of dynamic viscosity and density of the fluid, expressed in seconds\(^{-1}\).
# APPENDIX 5 – List of Weir Slurry Division Technical Bulletins

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