Chapter 7. Magnetic Materials and Magnetic Circuit Analysis

Topics to cover:

1) *Core Losses*
2) *Circuit Model of Magnetic Cores*
3) *A Simple Magnetic Circuit*
4) *Magnetic Circuital Laws*
5) *Circuit Model of Permanent Magnets*
6) *Calculation of Inductance, EMF, and Magnetic Energy*

Introduction

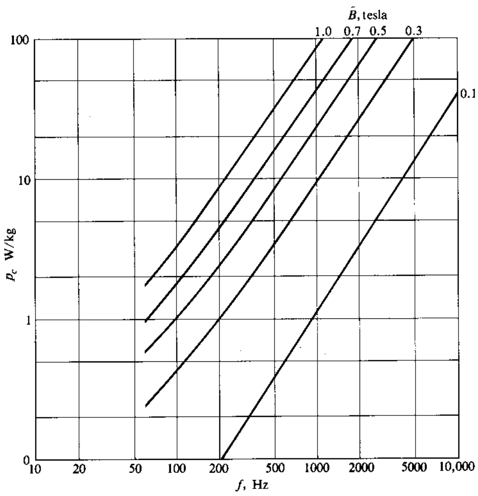
In general, magnetic materials can be classified as magnetically "soft" and "hard" materials. Soft materials are normally used as the magnetic core materials for inductors, transformers, actuators and rotating machines, in which the magnetic fields vary frequently, whereas hard materials, or permanent magnets, are used to replace magnetization coils for generating static magnetic fields in devices such as electric motors and actuators. The B-H relationships and hysteresis loops have been discussed earlier. In this chapter, we are going to examine the power losses in a soft magnetic core under an alternating magnetization, and further develop an electrical circuit model of a magnetic core with a coil.

For performance prediction of electromagnetic devices, magnetic field analysis is required. Analytical magnetic field analysis by the Maxwell's equations, however, has been shown very difficult for engineering problems owing to the fact that most practical devices are of complicated structures. Powerful numerical methods, such as the finite difference and finite element methods, are out of the scope of this subject. In this chapter, we introduce a simple method of magnetic circuit analysis based on an analogy to dc electrical circuits.

Soft Magnetic Materials under Alternating Excitations

<u>*Core Losses*</u>

Core losses occur in magnetic cores of ferromagnetic materials under alternating magnetic field excitations. The diagram on the right hand side plots the ***alternating core losses*** of M-36, 0.356 mm steel sheet against the excitation frequency. In this section, we will discuss the mechanisms and prediction of alternating core losses.



Alternating core loss of M36, 0.356 mm steel sheetat different excitation frequencies

As the external magnetic field varies at a ***very low rate*** periodically, as mentioned earlier, due to the effects of magnetic domain wall motion the *B-H* relationship is a hysteresis loop. The area enclosed by the loop is a power loss known as the hysteresis loss, and can be calculated by

$$P_{hyst} = \oint \mathbf{H} \bullet d\mathbf{B} \quad \text{(W/m}^3\text{/cycle) or (J/m}^3\text{)}$$

For magnetic materials commonly used in the construction of electric machines an approximate relation is

$$P_{hyst} = C_h f B_p^n \quad (1.5 < n < 2.5) \quad \text{(W/kg)}$$

where C_h is a constant determined by the nature of the ferromagnetic material, f the frequency of excitation, and B_p the peak value of the flux density.

Example:

A B-H loop for a type of electric steel sheet is shown in the diagram below. Determine approximately the hysteresis loss per cycle in a torus of 300 mm mean diameter and a square cross section of 50×50 mm.

Solution:

The are of each square in the diagram represents

$$(0.1 \text{ T}) \times (25 \text{ A/m}) = 2.5 \text{ (Wb/m}^2) \times \text{(A/m)} = 2.5 \text{ VsA/m}^3 = 2.5 \text{ J/m}^3$$

If a square that is more than half within the loop is regarded as totally enclosed, and one that is more than half outside is disregarded, then the area of the loop is

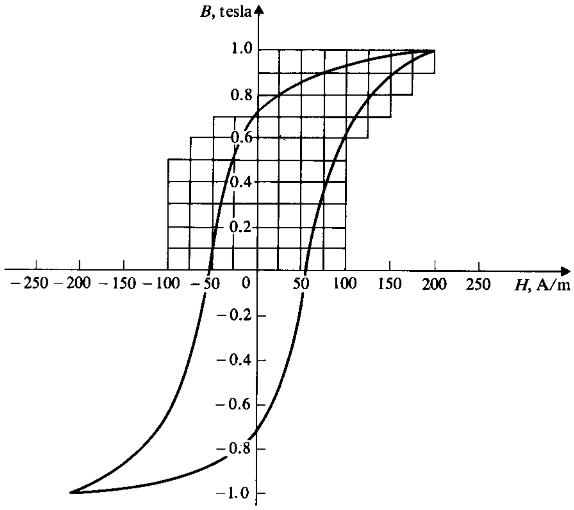
$$2 \times 43 \times 2.5 = 215 \text{ J/m}^3$$

The volume of the torus is

$$0.05^2 \times 0.3\pi = 2.36 \times 10^{-3} \text{ m}^3$$

Energy loss in the torus per cycle is thus

$$2.36 \times 10^{-3} \times 215 = 0.507 \text{ J}$$



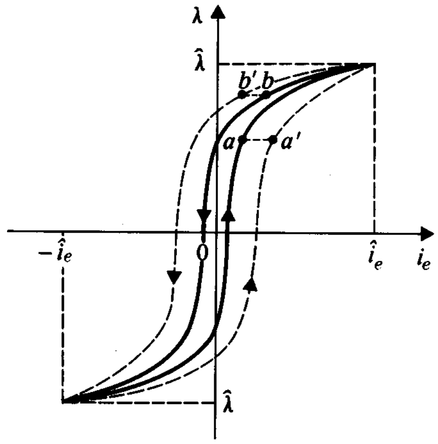
Hysteresis loop of M36 steel sheet

When the excitation field varies quickly, by the Faraday's law, an electromotive fore (*emf*) and hence a current will be induced in the conductor linking the field. Since most ferromagnetic materials are also conductors, eddy currents will be induced as the excitation field varies, and hence a power loss known as ***eddy current loss*** will be caused by the induced eddy currents. The resultant *B-H* or λ-i loop will be fatter due to the effect of eddy currents, as illustrated in the diagram below.

Under a sinusoidal magnetic excitation, the average eddy current loss in a magnetic core can be expressed by

$$P_{eddy} = C_e \left(f B_p \right)^2 \qquad \text{(W/kg)}$$

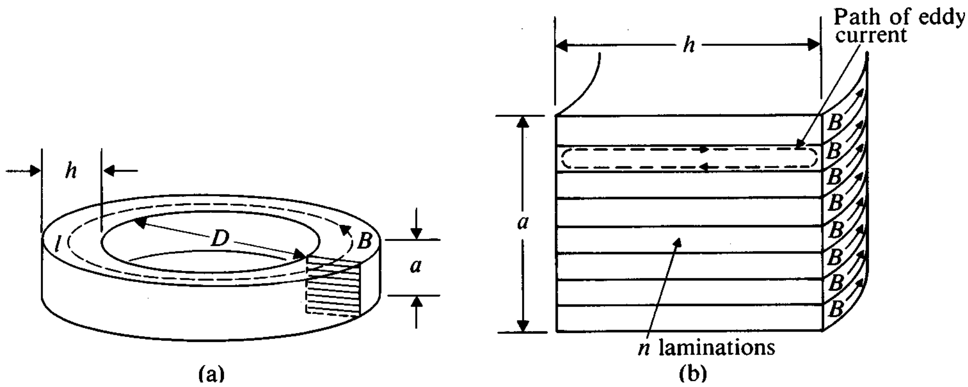
where C_e is a constant determined by the nature of the ferromagnetic material and the dimensions of the core.



Relationship between flux linkage and excitation current when eddy current is included (dashed line loop), where the solid line loop is the pure hysteresis obtained by dc excitation

Since the eddy current loss is caused by the induced eddy currents in a magnetic core., an effective way to reduce the eddy current loss is to increase the resistivity of the material. This can be achieved by adding Si in steel. However, too much silicon would make the steel brittle. Commonly used electrical steels contain 3% silicon.

Another effective way to reduce the eddy current loss is to use laminations of electrical steels. These electrical steel sheets are coated with electric insulation, which breaks the eddy current path, as illustrated in the diagram below.



Eddy currents in a laminated toroidal core

The above formulation for eddy current loss is obtained under the assumption of global eddy current as illustrated schematically in figure (a) of the following diagram. This is incorrect for materials with magnetic domains. When the excitation field varies, the domain walls move accordingly and local eddy currents are induced by the fluctuation of the local flux density caused by the domain wall motion as illustrated in figure (b) of the diagram below. The total eddy current caused by the local eddy currents is in general higher than that predicted by the formulation under the global eddy current assumption. The difference is known as the ***excess loss***. Since it is very difficult to calculate the total average eddy current loss analytically, by statistical analysis, it was postulated that for most soft magnetic materials under a sinusoidal magnetic field excitation, the excess loss can be predicted by

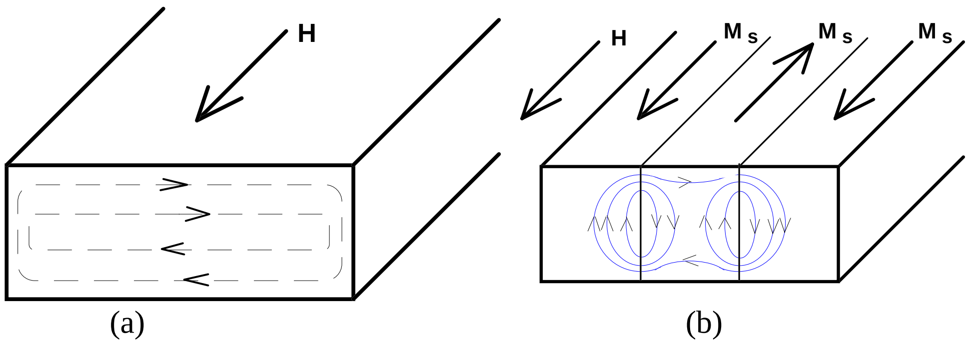
$$P_{ex} = C_{ex}\left(fB_p\right)^{3/2} \qquad \text{(W/kg)}$$

where C_{ex} is a constant determined by the nature of the ferromagnetic material.

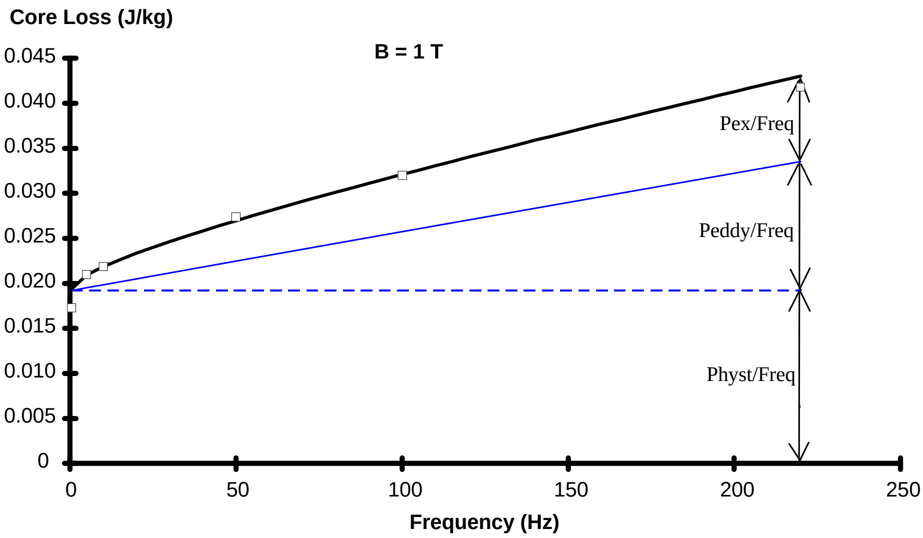
Therefore, the total core loss can be calculated by

$$P_{core} = P_{hyst} + P_{eddy} + P_{ex}$$

The diagram below illustrates the separation of alternating core loss of Lycore-140, 0.35 mm nonoriented sheet steel at 1 T. Using the formulas above, the coefficients of different loss components can be obtained by fitting the total core loss curves.



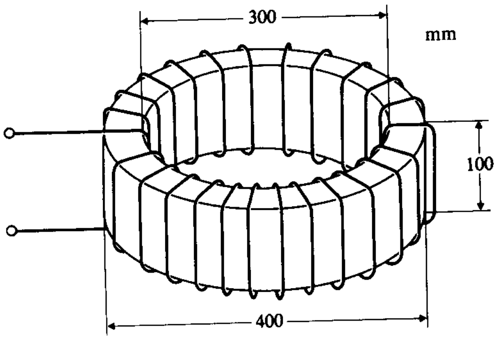
Eddy currents, (a) classical model, and (b) domain model



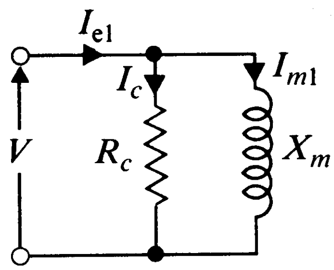
Separation of alternating core loss of Lycore-140 at B=1 T

Circuit Model of Magnetic Cores

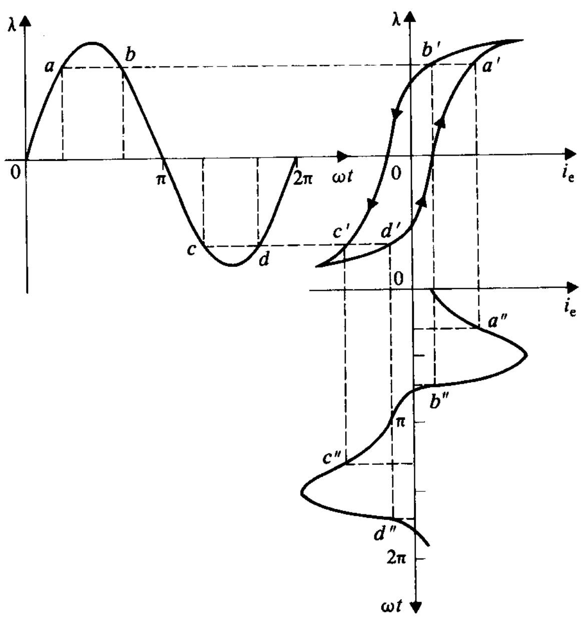
In the equivalent circuit of an electromagnetic device, the circuit model of the magnetic core is an essential part. Consider a magnetic core with a coil of N turns uniformly wound on it. As illustrated below, under an sinusoidal voltage (flux likage) excitation, the corresponding excitation current is nonsinusoidal due to the nonlinear *B*-*H* relationship of the core. When only the fundamental component of the current is considered, however, the relationship between the phasors of voltage and current can be determined by a resistor (equivalent resistance of the core loss) in parallel of an lossless indutor (self inductance of the coil) as illustrated in the diagram below.



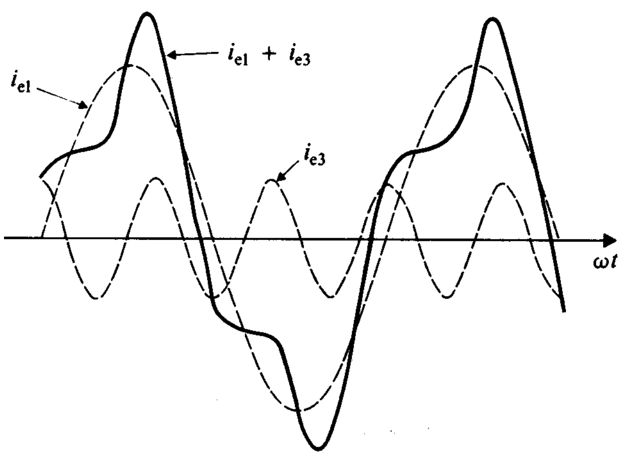
Coil of *N* turns with a magnetic core



Circuit model of magnetic cores



Excitation current corresponding to a sinusoidal voltage excitation



Fundamental and third harmonic in the excitation current

A Simple Magnetic Circuit

Consider a simple structure consisting of a current carrying coil of N turns and a magnetic core of mean length l_c and a cross sectional area A_c as shown in the diagram below. The permeability of the core material is μ_c. Assume that the size of the device and the operation frequency are such that the displacement current in Maxwell's equations are negligible, and that the permeability of the core material is very high so that all magnetic flux will be confined within the core. By Ampere's law,

$$\oint_C \mathbf{H} \bullet d\mathbf{l} = \oint_S \mathbf{J} \bullet d\mathbf{a}$$

we can write

$$H_c l_c = Ni$$

where H_c is the magnetic field strength in the core, and Ni the ***magnetomotive force***. The magnetic flux through the cross section of the core can expressed as

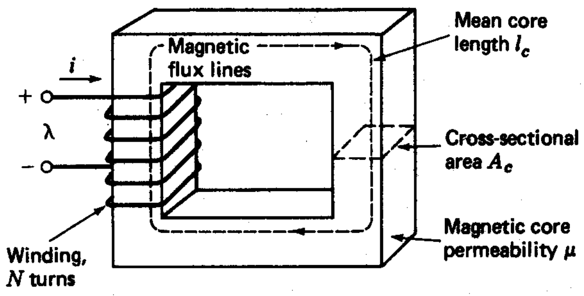
$$\phi_c = B_c A_c$$

where ϕ_c is the flux in the core and B_c the flux density in the core. The constitutive equation of the core material is

$$B_c = \mu H_c$$

Therefore, we obtain

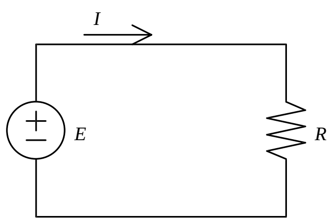
$$\phi_c = \frac{Ni}{l_c/(\mu_c A_c)} = \frac{F}{R_c}$$



A simple magnetic circuit

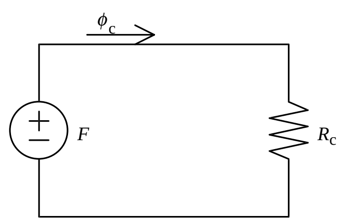
If we take the magnetic flux ϕ_c as the "current", the magnetomotive force $F=Ni$ as the "emf of a voltage source", and $R_c=l_c/(\mu_c A_c)$ (known as the ***magnetic reluctance***) as the "resistance" in the magnetic circuit, we have an analog of ***Ohm's law*** in electrical circuit theory.

Electric Circuit



$$I = \frac{E}{R}$$

Magnetic Circuit



$$\phi_c = \frac{F}{R_c}$$

Magnetic Circuital Laws

Consider the magnetic circuit in the last section with an air gap of length l_g cut in the middle of a leg as shown in figure (a) in the diagram below. As they cross the air gap, the magnetic flux lines bulge outward somewhat as illustrate in figure (b). The effect of the ***fringing*** field is to increase the effective cross sectional area A_g of the air gap. By Ampere's law, we can write

$$F = Ni = H_c l_c + H_g l_g$$

where $$H_c l_c = \frac{B_c}{\mu_c} l_c = \frac{\phi_c}{\mu_c A_c} l_c = \phi_c R_c$$

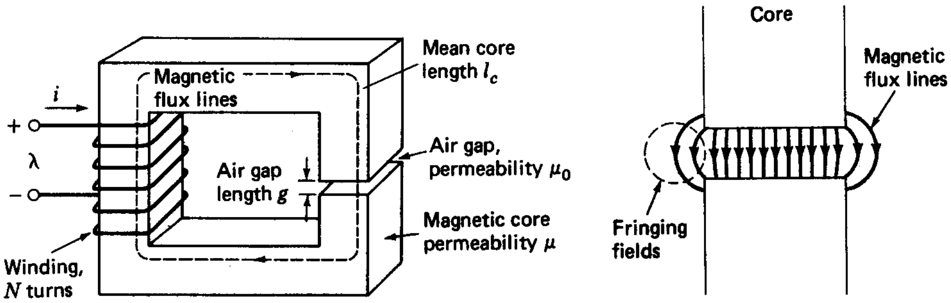
and $$H_g l_g = \frac{B_g}{\mu_o} l_g = \frac{\phi_g}{\mu_o A_g} l_g = \phi_g R_g$$

According to Gauss' law in magnetics,

$$\oint_S \mathbf{B} \bullet d\mathbf{a} = 0$$

we know

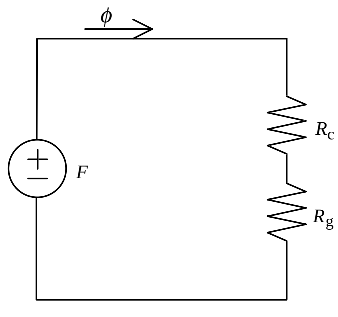
$$\phi_c = \phi_g = \phi$$



A simple magnetic circuit with an air gap

Therefore,

$$F = \left(R_c + R_g\right)\phi$$



Series magnetic circuit

That is, the above magnetic circuit with an air gap is analogous to a series electric circuit. Further, if we regard $H_c l_c$ and $H_g l_g$ as the "voltage drops" across the reluctance of the core and airgap respectively, the above equation from Ampere's law can be interpreted as an analog to the ***Kirchhoff's voltage law*** (KVL) in electric circuit theory, or

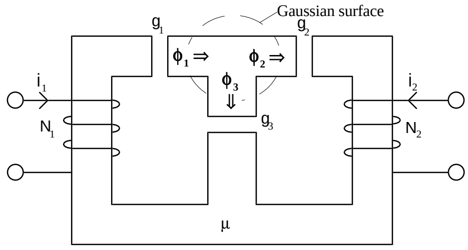
$$\sum R_k \phi_k = \sum F_k$$

The ***Kirchhoff's current law*** (KCL) can be derived from the Gauss' law in magnetics. Consider a magnetic circuit as shown below. When the Gauss' law is applied to the T joint in the circuit, we have

$$\sum_{k=1}^{3} \phi_k = 0$$

or in general,

$$\sum_{k=1}^{n} \phi_k = 0$$



Magnetic circuit of T joints

Having derived the Ohm's law, KVL and KCL in magnetic circuits, we can solve very complex magnetic circuits by applying these basic laws. All electrical dc circuit analysis techniques, such as mesh analysis and nodal analysis, can also be applied in magnetic circuit analysis.

For nonlinear magnetic circuits where the nonlinear magnetization curves need to be considered, the magnetic reluctance is a function of magnetic flux since the permeability is a function of the magnetic field strength or flux density. Numerical or graphical methods are required to solve nonlinear problems.

Magnetic Circuit Model of Permanent Magnets

Permanent magnets are commonly used to generate magnetic fields for electromechanical energy conversion in a number of electromagnetic devices, such as actuators, permanent magnet generators and motors. As mentioned earlier, the characteristics of permanent magnets are described by demagnetization curves (the part of hysteresis loop in the second quadrant). The diagram below depicts the demagnetization curve of five permanent magnets. It can be seen that the demagnetization curves of some most commonly used permanent magnets: Neodymium Iron Boron (NdFeB), Samarium Cobalt, and Ceramic 7 are linear. For the convenience of analysis, we consider the magnets with linear demagnetization curves first.

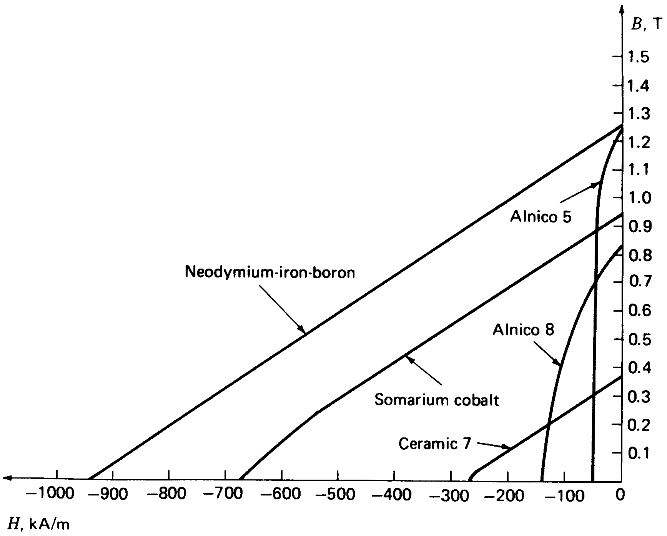
Consider a piece of permanent magnet of a uniform cross sectional area of A_m and a length l_m. The demagnetization curve of the magnet is a straight line with a coercive force of H_c and a remanent flux density of B_r as shown below. The demagnetization curve can be expressed analytically as

$$B_m = \frac{B_r}{H_c}\left(H_m + H_c\right) = \mu_m\left(H_m + H_c\right)$$

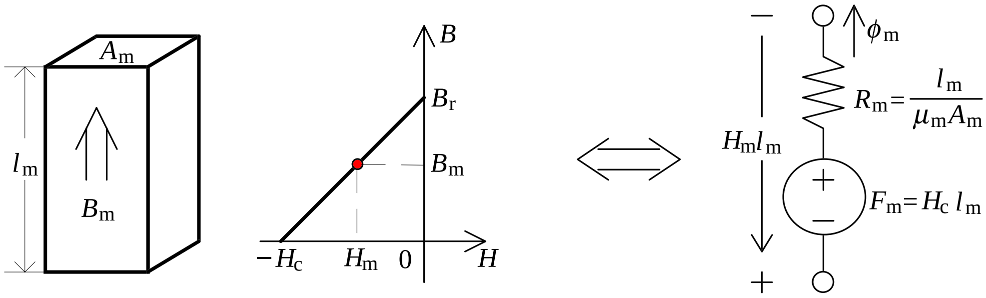
where $\mu_m = B_r/H_c$ is the permeability of the permanent magnet, which is very close to μ_o, the permeability of free space. For a NdFeB magnet, $\mu_m = 1.05\mu_o$.

The magnetic "voltage drop" across the magnet can be expressed as

$$H_m l_m = \left(\frac{B_m}{\mu_m} - H_c\right) l_m = \frac{l_m}{\mu_m A_m}\phi_m - H_c l_m = R_m \phi_m - F_m$$



Demagnetization curves of permanent magnets



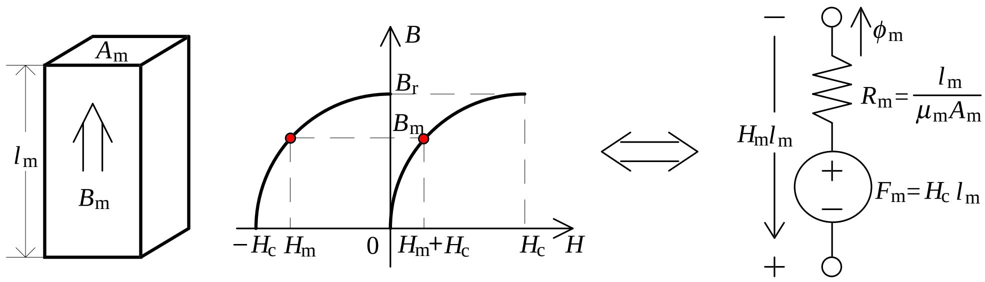
Magnetic circuit model of a magnet with linear demagnetization curve

where $R_m = \dfrac{l_m}{\mu_m A_m}$ is the reluctance and $F_m = H_c l_m$ the magnetomotive force (“voltage source”) of the magnet. It should be noted that in the magnet, $\mathbf{B}_m$ and $\mathbf{H}_m$ are in opposite directs.

For a magnet with a nonlinear demagnetization curve, the above magnetic circuit model is still valid, except that the magnetic permeability becomes

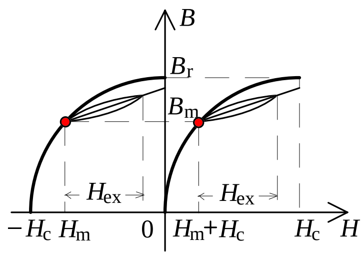
$$\mu_m = \frac{B_m}{H_m + H_c}$$

which is a function of the magnetic field in the magnet. Notice that H_m is a negative value since it is in the opposite direction of $\mathbf{B}_m$. The derivation for the magnetic circuit model of a nonlinear magnet is illustrated graphically by the diagram below.



Magnetic circuit model of a magnet with nonlinear demagnetization curve

It should also be understood that the operating point (H_m,B_m) will not move along the nonlinear demagnetization curve if a small (such that the magnet will not be demagnetized) periodic external magnetic field is applied to the magnet. Instead, the operating point will move along a minor loop or simply a straight line (center line of the minor loop) as illustrated in the diagram on the right hand side.



Movement of operating point of a nonlinear magnet under an external field H_{ex}

Inductance

Consider a two coil magnetic system as shown below. The magnetic flux linkage of the two coils can be express as

$$\lambda_1 = \lambda_{11} + \lambda_{12} \qquad \text{and} \qquad \lambda_2 = \lambda_{21} + \lambda_{22}$$

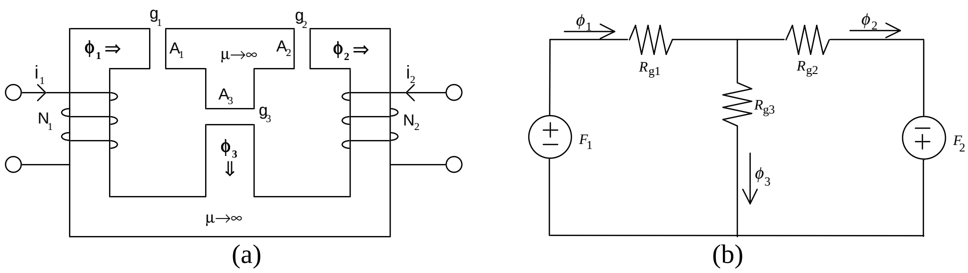
where the first subscript indicates the coil of flux linkage and the second the coil carrying current. By defining the self and mutual inductances of the two coils as

$$L_{jk} = \frac{\lambda_{jk}}{i_k} \qquad (j=1,2 \text{ and } k=1,2)$$

where L_{jk} is the self inductance of the *j*th coil when $j=k$, the mutual inductance between the *j*th coil and the *k*th coil when $j\neq k$, and $L_{jk} = L_{kj}$, the flux linkages can be expressed as

$$\lambda_1 = L_{11}i_1 + L_{12}i_2 \qquad \text{and} \qquad \lambda_2 = L_{21}i_1 + L_{22}i_2$$

The above definition is also valid for a *n* coil system. For a linear magnetic system, the above calculation can be performed by switching on one coil while all other coils are switched off such that the magnetic circuit analysis can be simplified. This is especially significant for a complex magnetic circuit. For a nonlinear magnetic system, however, the inductances can only be calculated by the above definition with all coils switched on.



Magnetic circuit of a two coil system

Electromotive Force

When a conductor of length *l* moves in a magnetic field of flux density **B** at a speed **v**, the induced electromotive force (*emf*) can be calculated by

$$\mathbf{e} = l\mathbf{v} \times \mathbf{B}$$

For a coil linking a time varying magnetic field, the induced *emf* can be calculated from the flux linkage of the coil by

$$e_k = \frac{d\lambda_k}{dt} = \sum_{j=1}^{n} \frac{d\lambda_{kj}}{dt} = \sum_{j=1}^{n} L_{kj} \frac{di_j}{dt} \qquad (k=1,2,\ldots n)$$

Magnetic Energy

In terms of inductance, the magnetic energy stored in an *n* coil system can be expressed as

$$W_f = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} \lambda_{jk} i_j = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} \frac{\lambda_{jk}\lambda_{kj}}{L_{jk}} = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} L_{jk} i_j i_k$$

Exercises

1. Show that the hysteresis energy loss per unit volume per cycle due to an AC excitation in an iron ring is equal to the area of the B-H loop, i.e.

   $$\oint HdB$$

   The hysteresis loop for a certain iron ring is drawn in terms of the flux linkage λ of the excitation coil and the excitation current i_m to the following scales

   on the excitation current i_m axis: 1 cm = 500 A

   on the flux linkage λ axis: 1 cm = 100 μWb

   The area of the hysteresis loop is 50 cm^2 and the excitation frequency is 50 Hz. Calculate the hysteresis power loss of the ring.

   Answer: 125 W

2. A coils of 200 turns is wound uniformly over a wooden ring having a mean circumference of 600 mm and a uniform cross sectional area of 500 mm^2. If the current through the coil is 4 A, calculate:
   (a) the magnetic field strength,
   (b) the flux density, and
   (c) the total flux

   Answer: 1333 A/m, 1675×10^{-6} T, 0.8375 μWb

3. A mild steel ring having a cross sectional area of 500 m^2 and a mean circumference of 400 mm has a coil of 200 turns wound uniformly around it. Calculate:
   (a) the reluctance of the ring and
   (b) the current required to produce a flux of 800 μWb in the ring. (Given that μ_r is about 380).

   Answer: 1.677×10^{6} A/Wb, 6.7 A

4. Fig.Q4 shows an iron circuit with a small air gap cut in it. A 6000 turn coil carries a current I=20 mA which sets up a flux within the iron and across the air gap. If the iron cross section is 0.8×10^{-4} m^2, the mean length of flux path in iron is 0.15 m, μ_r=800 in iron and air gap length is 0.75 mm, calculate the air gap flux density. It may be assumed that the flux lines flow straight across the air gap, i.e. air gap cross section is also 0.8×10^{-4} mm^2.
Answer: 0.16 T

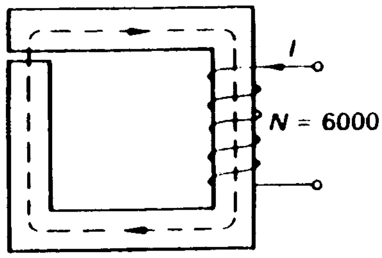


Fig.Q4

5. A magnetic circuit is made of mild steel arranged as in Fig.Q5. The center limb is wound with 500 turns and has a cross sectional area of 800 mm^2. Each of the outer limbs has a cross sectional area of 500 mm^2. The air gap has a length of 1 mm. Calculate the current required to set up a flux of 1.3 mWb in the center limb, assuming no magnetic leakage and fringing. The mean lengths of the various magnetic paths are shown on the diagram. (Use the given B-H curve).
Answer: 4 A

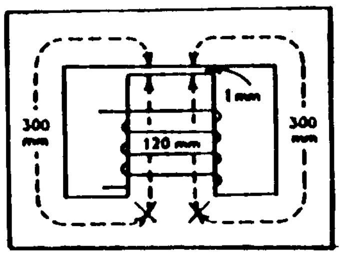


Fig.Q5

6. A magnetic circuit is made up of steel laminations shaped as in Fig.Q6. The width of the iron is 40 mm and the core is built up to a depth of 50 mm, of which 8 percent is taken up by insulation between the laminations. The gap is 2 mm long and the effective area of the gap is 2500 mm^2. The coil is wound with 800 turns. If the leakage factor (the ratio of the total flux linking the coil over the air gap flux) is 1.2, calculate the magnetizing current required to produce a flux of 0.0025 Wb across the air gap. (Use the given B-H curve).
Answer: 5 A

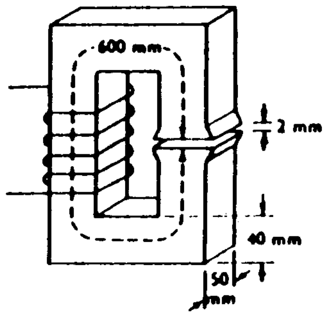
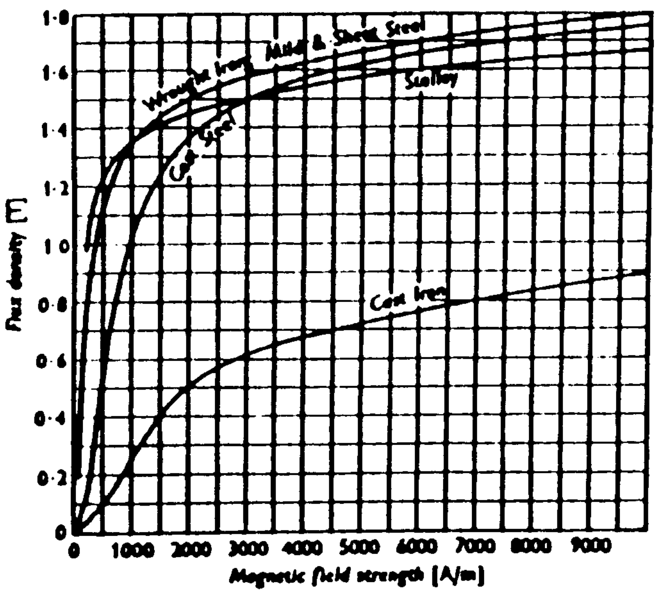


Fig.Q6



7. It is desired to achieve a time varying magnetic flux density in the air gap of the magnetic circuit of Fig.Q7(a) of the form

$$B_g = B_0 + B_1 \sin\omega t$$

where B_0=0.5 T and B_1=0.25 T. The dc field B_0 is to be created by a NdFeB permanent magnet, whereas the time varying field is to be created by a time varying current. Assume the permeability of the iron is infinite and neglect the fringing effect.

(a) For the air gap dimensions given in Fig.Q7(a), find the magnet length *d* if the magnet area A_m equals the air gap area A_g. Fig.Q7(b) gives the demagnetization curve of NdFeB permanent magnet.

(b) Find the excitation current required to achieve the desired time varying air gap flux density.

Answer: 2.64 (mm), i = 5.28sinωt (A)

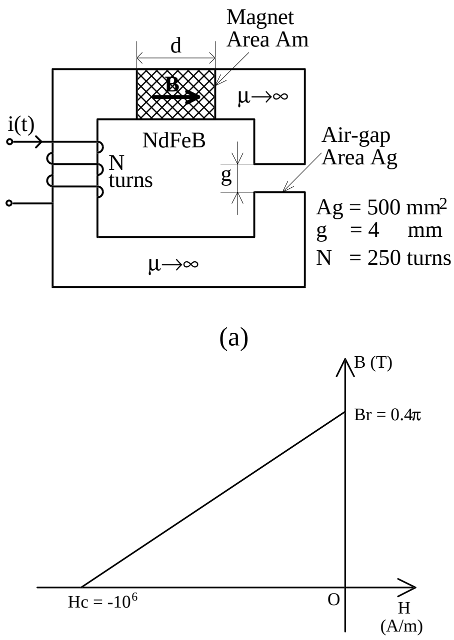


Fig.Q7 (a) Magnetic circuit of Problem 1, (b) Demagnetization curve of permanent magnet NdFeB

8. Fig.Q8 shows a magnetic circuit with air gaps $\mathbf{g_1} = \mathbf{g_2} = \mathbf{g_3}$ = 1 mm and coils $\mathbf{N_1}$ = 100 turns and $\mathbf{N_2}$ = 200 turns. The cross sectional area **A** of the circuit is 200 mm^2. Assume the permeability of the core material approaches infinity and the fringing effect is negligible.

Calculate:

(a) the self and mutual inductances;

(b) the total magnetic energy stored in the system, if the currents in the coils are $\mathbf{i_1} = \mathbf{i_2}$ = 1 A;

(c) the mutual inductance between $\mathbf{N_1}$ and $\mathbf{N_2}$, if the air gap $\mathbf{g_3}$ is closed.

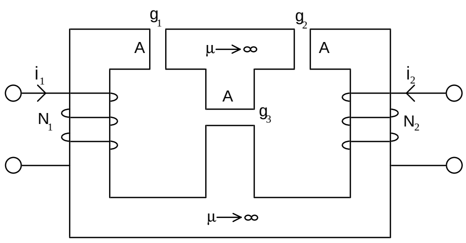


Fig.Q8 Magnetic circuit of Problem 7

Answer: 1.676 mH, 6.702 mH, 1.676 mH, 5.865×10^{-3} J, 0