UNIVERSITY OF TECHNOLOGY, SYDNEY
FACULTY OF ENGINEERING

48531 Electromechanical Systems

Magnetostatics

Topics to cover:

1. Introduction
2. Flux Density
3. Biot-Savart Law
4. Field Strength
5. Scalar Potential
6. Ampere's Circuital Law
7. Magnetic Materials
8. Inductances
9. Magnetic Energy
10. Magnetic Forces

Introduction

- In 1785 ***Coulomb*** developed a law for the ***force between magnetic poles*** which was precisely similar to the law he had stated for the force between electric charges. Based on this law the modern western theory of magnetostatics developed.
- In 1820 ***Oersted*** showed that ***magnetic effects could be produced by an electric current*** flowing in a circuit. The nature of fields generated this way was further explored by ***Ampere***. As earlier development was based on permanent magnets, Ampere expressed his results in terms of a distribution of magnetic poles that was equivalent to the current carrying circuit producing the magnetic effect.
- A fundamental difference between electrostatics and magnetostatics is that while positive and negative electric charges exist separately, north and south magnetic poles always occur in pairs. (Some researchers are still searching for a isolated magnetic pole.)

Magnetic Flux and Flux density

The total electromagnetic force acting on a charge q moving at speed $\mathbf{u}$ is

$$\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m = q\mathbf{E} + q\mathbf{u} \times \mathbf{B}$$

where $\mathbf{F}_e$ and $\mathbf{F}_m$ are the electric and magnetic forces, and $\mathbf{B}$ the magnetic flux density (Wb/m^2 or T) defined by $\mathbf{F}_m / q = \mathbf{u} \times \mathbf{B}$

The magnetic flux over a cross section is then defined as

$$\phi = \int_S \mathbf{B} \bullet d\mathbf{a}$$

The Gauss' law in magnetics (closed loop magnetic flux lines)

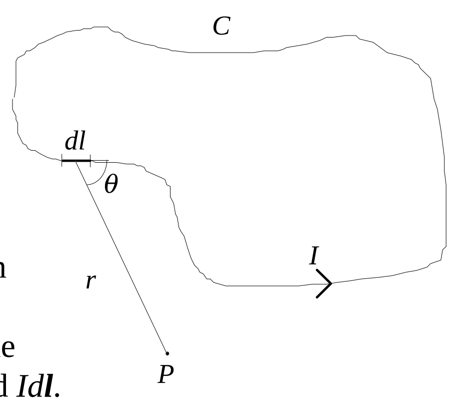
$$\oint_S \mathbf{B} \bullet d\mathbf{a} = 0$$

Biot-Savart Law

The magnetic flux density produced by a current I in a circuit C can be determined by

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi r^2} d\boldsymbol{l} \times \hat{\mathbf{r}}$$

$$\mathbf{B} = \frac{\mu_0 I}{4\pi} \oint_C \frac{d\boldsymbol{l} \times \hat{\mathbf{r}}}{r^2}$$

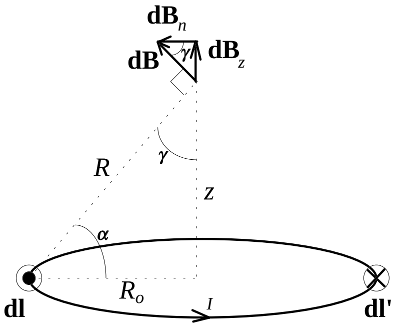


where $d\mathbf{B}$ is the flux density due to an elemental current $I\boldsymbol{dl}$, $\mu_o = 4\pi \times 10^{-7}$ the permeability of free space, and $\mathbf{r}$ is the distance between the field point P and $I\boldsymbol{dl}$.

Biot-Savart Law - Example

Apply the law of Biot and Savart the axial flux density of a circular current.

Solution:



Consider the above drawing. By the Biot-Savart the contribution of a small segment of the circuit (***dl***) to the field flux on the axis is given by $dB = \dfrac{\mu_0 I dl}{4\pi R^2}$

as the current element is at right angles to the radius vector (θ=90^o). The element ***dl'*** opposite ***dl*** on the circuit contributes a component that cancels $\mathbf{dB}_n$. Thus the final contribution of each element ***dl*** points up (z-directed) with magnitude

$$dB_z = \frac{\mu_0 I dl}{4\pi R^2}\cos\alpha$$

Biot-Savart Law - Example

Summing the contributions of all dl leads to

$$B = B_z = \int dB_z = \frac{\mu_0 I \cos\alpha}{4\pi R^2}\oint dl$$

The line integration around the loop is $2\pi R_o$, and so

$$B = B_z = \frac{\mu_0 I R_o \cos\alpha}{2R^2}$$

Noting that $\cos\alpha = R_0 / R$ and $A = \pi R_0^2$

Therefore, we can arrive an alternative form such as

$$\mathbf{B} = \frac{\mu_0 I A}{2\pi R^3}\hat{\mathbf{z}}$$

Magnetic Field Strength

If we are only concerned with magnetic effects in free space then a knowledge of the magnetic flux density, **B,** would be enough to describe the magnetic field. Later, when magnetised bodies are considered it will be necessary to use a second vector, the **magnetic field strength**, **H**. In free space **H** is defined by the equation

$$\mathbf{B} = \mu_0 \mathbf{H}$$

The Biot-Savart law could be used to find **H**. **H** is sometimes called the magnetising force. Its units are ***amperes per meter Am^{-1}***.

Magnetic Scalar Potential and Ampere's Circuital Law

The ***magnetic scalar potential***, *U*, is defied as

$$U_{BA} = -\int_A^B \mathbf{H} \bullet d\mathbf{l} \qquad \text{(A)}$$

Ampere's law states that the sum of the magnetic potentials around a contour, *C*, surrounding a current, *I*, will equal *I*. That is

$$\oint_C \mathbf{H} \bullet d\mathbf{l} = I$$

Ampere's Circuital Law - Example

Calculate the flux density R_o distance away from an infinitely long conductor carrying current I as in the diagram below by the Ampere's law and the Biot-Savart law.

Solution:

By Ampere's law, we have

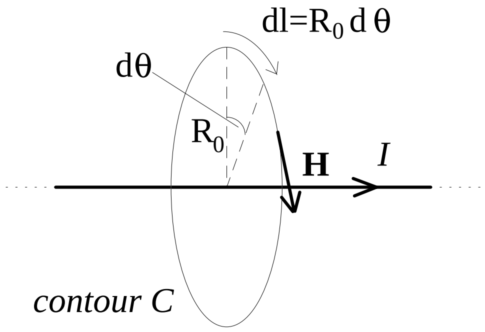
$$\int_C \mathbf{H} \bullet d\boldsymbol{l} = 2\pi R_o H = I$$

or

$$H = \frac{I}{2\pi R_o}$$

Therefore,

$$B = \mu_o H = \frac{\mu_o I}{2\pi R_o}$$



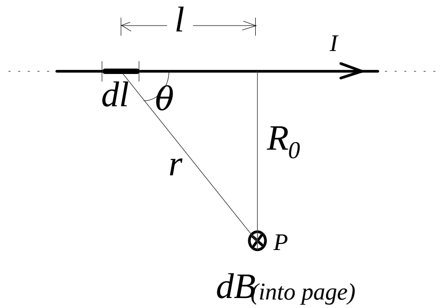
Ampere's Circuital Law - Example

By the Biot-Savart law, we may write

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

and

$$B = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty}\frac{\sin\theta\, dl}{r^2}$$

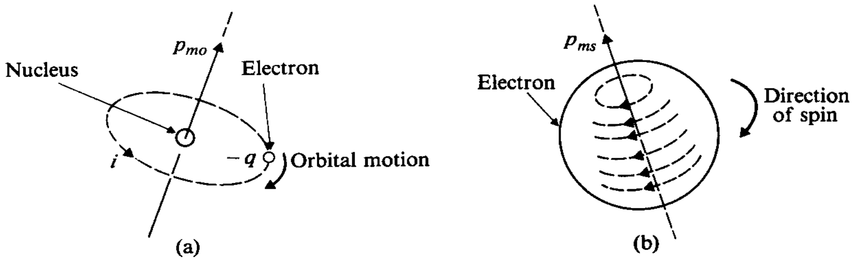


Now $R_o = l\tan\theta$, where R_o is constant. By differentiation we can obtain $\cos\theta\sin\theta dl = l d\theta$, and as $r\cos\theta = l$ and $r\sin\theta = R_o$, we can write

$$B = \frac{\mu_0 I}{4\pi R_0}\int_0^{\pi}\sin\theta\, d\theta = \frac{\mu_0 I}{2\pi R_0}$$

Magnetic Properties of Materials

In the atoms of a material, the orbiting electrons cause circulating currents and form microscopic magnetic dipoles. In addition, both the electrons and the nucleus of an atom spin on their own axes with certain magnetic dipole moments. The magnetic moment of a spinning nucleus is usually negligible in comparison to that of an orbiting and spinning electron because of the much larger mass and lower angular velocity of the nucleus. The diagram below illustrates schematically the orbital motion and the spin of an electron



Magnetic Properties of Materials

In the absence of an external magnetic field the magnetic dipoles of the atoms of most materials (except permanent magnets) have random orientations, resulting in no net magnetic moment. The application of an external magnetic field cause both an alignment of magnetic moments of the spinning electrons and an induced magnetic moment due to a charge in orbital motion of electrons. To obtain a formula for determining the quantitative change in the magnetic flux density caused by the presence of a magnetic material, we let $\mathbf{m}_k$ be the magnetic dipole moment of an atom. If there are n atoms per unit volume, we define a ***magnetization vector***, **M**, as

$$\mathbf{M} = \lim_{\Delta v \to 0}\frac{\sum_{k=1}^{n\Delta v}\mathbf{m}_k}{\Delta v}$$

which is the volume density of magnetic dipole moment in (A/m).

Magnetic Properties of Materials

In a magnetized material, the magnetic flux density **B** has two components contributed respectively by the external magnetic field and the magnetization:

$$\mathbf{B} = \mu_o(\mathbf{H} + \mathbf{M})$$

When the magnetic properties of the medium are linear and isotropic, the magnetization is directly proportional to the magnetic field strength:

$$\mathbf{M} = \chi_m \mathbf{H}$$

where χ_m is a dimensionless quantity known as the ***magnetic susceptibility***. Therefore,

$$\mathbf{B} = \mu_o(1+\chi_m)\mathbf{H} = \mu_o\mu_r\mathbf{H} = \mu\mathbf{H}$$

where μ_r is the ***relative permeability***, and μ the ***absolute permeability*** (or sometimes just permeability) in henry per meter or H/m.

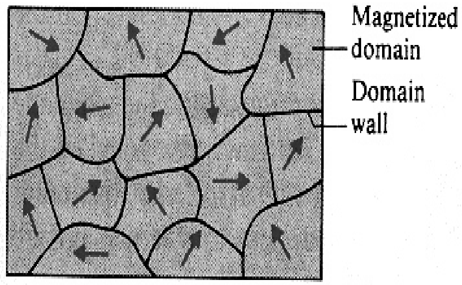
Magnetic Properties of Materials

All materials can be roughly classified into three main groups in accordance with their μ_r values. A material is said to be

Diamagnetic, if $\mu_r \approx 1$ and $\mu_r < 1$ (χ_m is a very small negative number), or
Paramagnetic, if $\mu_r \approx 1$ and $\mu_r > 1$ (χ_m is a very small positive number), or
Ferromagnetic, if $\mu_r >> 1$ (χ_m is a large positive number).

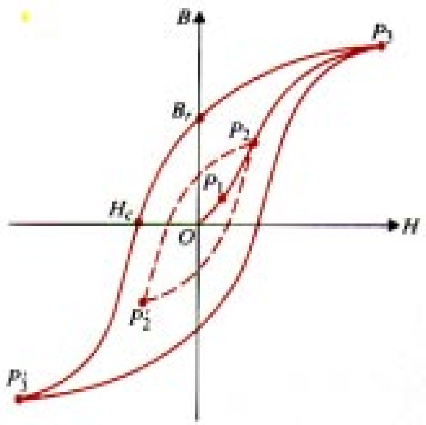
The magnetization of ***ferromagnetic*** materials can be many orders of magnitude larger than that of paramagnetic substances.

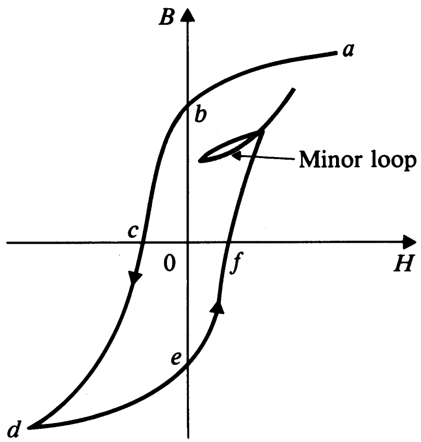
Ferromagnetism can be explained in terms of magnetized ***domains***.



Magnetic Properties of Materials

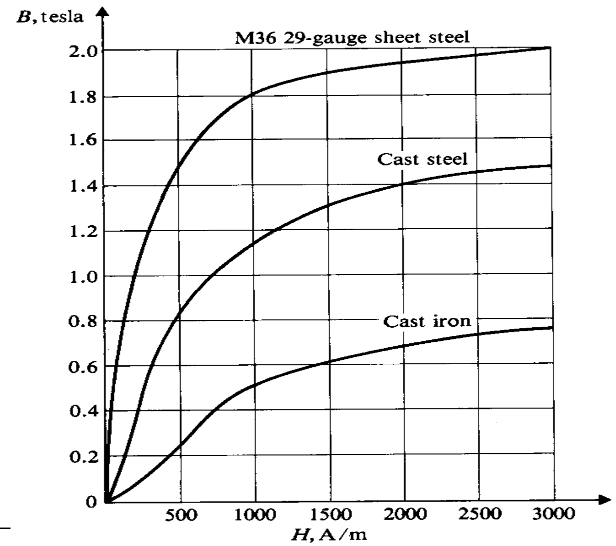
- *Magnetic Hysteresis*





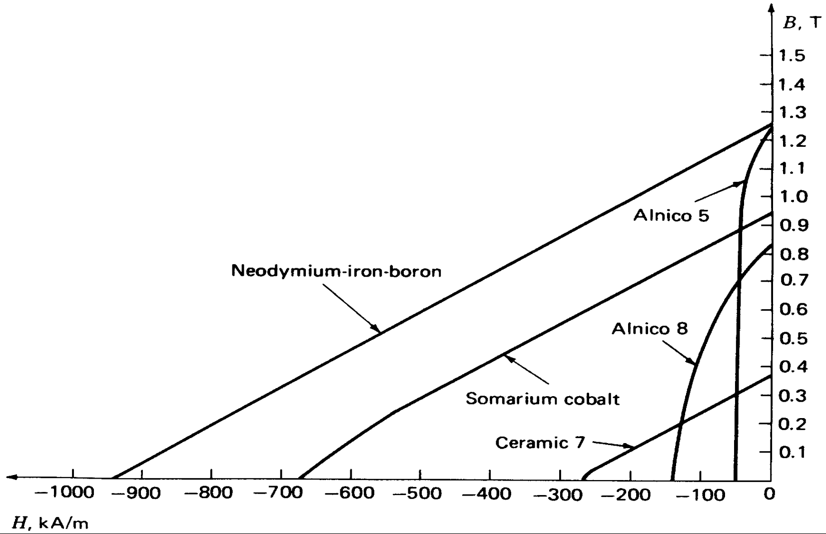
Magnetic Properties of Materials

- *Normal Magnetization Curves*



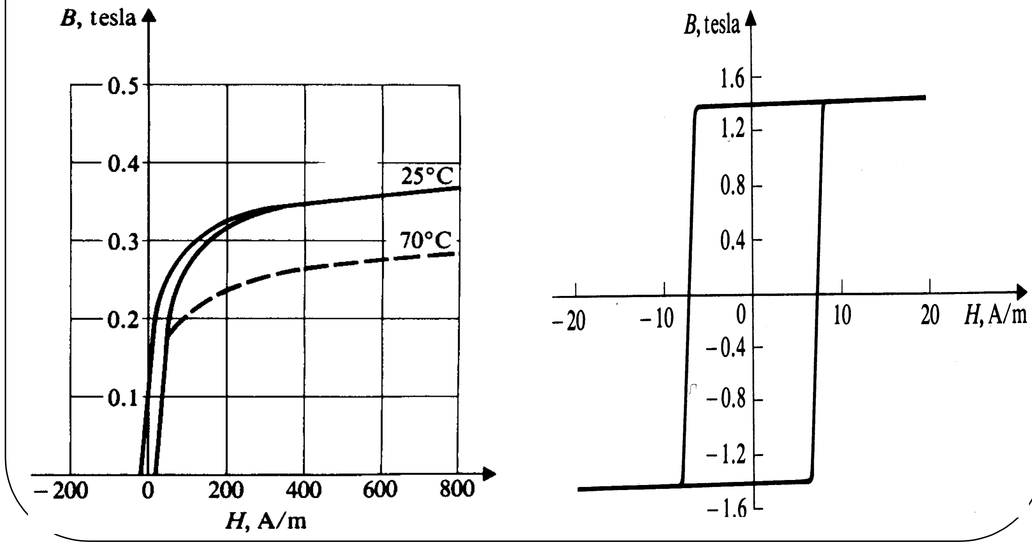
Magnetic Properties of Materials

- Demagnetization Curves of Permanent Magnets



Magnetic Properties of Materials

- Hystersis Loops of Soft Ferrites & Recording Media



Inductances

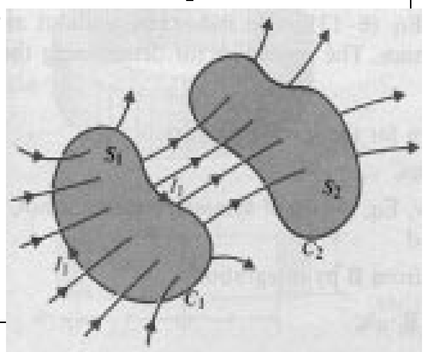
Consider two neighboring coils, C_1 (of N_1 turns) and C_2 (of N_2 turns) bounding surfaces S_1 and S_2 as shown in the diagram. If a current I_1 flows in C_1, a magnetic field $\mathbf{B}_1$ will be created. All flux links C_1 and some of the flux will link C_2, and the flux linkages can be calculated by

$$\lambda_{11} = N_1\phi_{11} = N_1\int_{S_1}\mathbf{B}_1 \bullet d\mathbf{a} \quad \text{and} \quad \lambda_{21} = N_2\phi_{21} = N_2\int_{S_2}\mathbf{B}_1 \bullet d\mathbf{a}$$

From the Biot-Savart law, we know that the flux density and hence the flux linkage is proportional to the current I_1. We write

$$\lambda_{11} = L_{11}I_1 \quad \text{and} \quad \lambda_{21} = L_{21}I_1$$

where L_{11} and L_{12} are defined as the self inductance of coil 1 and the mutual inductance between the two coils, respectively.



Inductances *- cont.*

The self inductance of coil 2 can be obtained similarly by introducing a current in it. It can be shown that the mutual inductances calculated from both sides are equal or L_{21}=L_{12}.

Neumann formula

$$L_{12} = \frac{\mu_o}{4\pi}\oint_{C_1}\oint_{C_2}\frac{d\mathbf{l}_1 \bullet d\mathbf{l}_2}{R}$$

where N_1 and N_2 are absorbed in the contour integrals over C_1 and C_2, and R is the distance between $d\mathbf{l}_1$ and $d\mathbf{l}_2$.

Magnetic Energy *- Formulations*

In terms of field quantities the energy stored in a magnetic field can be determined by

$$W_m = \frac{1}{2}\int_{V'} \mathbf{H} \bullet \mathbf{B} dv'$$

In a system consists of *n* coils, the magnetic energy can be expressed in terms of inductance as

$$W_m = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} L_{jk} I_j I_k$$

Magnetic Force

The force acting on a current carrying conductor C can be derived directly from the force acting on moving charges as

$$\mathbf{F}_m = I\int_C d\boldsymbol{l} \times \mathbf{B}$$

For a single conductor in a uniform magnetic field, we have

$$\mathbf{F}_m = I\boldsymbol{l} \times \mathbf{B}$$

Summary

- Magnetic Flux and Gauss' Law: $\phi = \int_S \mathbf{B} \bullet d\mathbf{a}$, $\oint_S \mathbf{B} \bullet d\mathbf{a} = 0$
- Biot-Savart Law: $\mathbf{B} = \frac{\mu_0 I}{4\pi}\oint_C \frac{d\mathbf{l} \times \hat{\mathbf{r}}}{r^2}$, $\mu_o = 4\pi \times 10^{-7}$
- Field Strength and Ampere's Law: $\mathbf{B} = \mu_o \mu_r \mathbf{H}$, $\oint_C \mathbf{H} \bullet d\boldsymbol{l} = I$
- Self and Mutual Inductance: $L_{11} = \frac{\lambda_{11}}{I_1}$, $L_{12} = \frac{\lambda_{12}}{I_2}$
- Magnetic Energy: $W_m = \frac{1}{2}\int_{V'} \mathbf{H} \bullet \mathbf{B} dv'$, $W_m = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} L_{jk} I_j I_k$
- Magnetic Force: $\mathbf{F}_m = I\int_C d\boldsymbol{l} \times \mathbf{B}$