INDUCTION MOTORS

1. The speed of the rotating magnetic field

\[ \omega_{\text{syn}} = \frac{2\pi f_1}{P/2} \quad \text{(rad/s)} \]

or

\[ n_{\text{syn}} = \frac{60 f_1}{P/2} \quad \text{(rev/min)} \]

where \( f_1 \) is the frequency of the stator current, and \( P \) the number of poles.

2. The rotor speed of an induction machine is generally different from the speed of the rotating magnetic field. The percentage difference of speed, known as slip, is defined as

\[ s = \frac{\omega_{\text{syn}} - \omega_m}{\omega_{\text{syn}}} = \frac{n_{\text{syn}} - n}{n_{\text{syn}}} \]

where \( \omega_m \) and \( n \) are the speed of the rotor in rad/s and rev/min, respectively.

3. The induced emf in one phase of the stator winding is

\[ E_1 = 4.44 f_1 N_1 k_{w1} \Phi_m \]

and the emf in the rotor winding

\[ E_{2s} = 4.44 f_2 N_2 k_{w2} \Phi_m \]

where \( f_1 \) and \( f_2 \) are the frequencies of the emf's in the stator and rotor windings, \( N_1 k_{w1} \) and \( N_2 k_{w2} \) the effective number of turns of the stator and rotor windings, and \( \Phi_m \) the magnitude of the rotating magnetic flux.
Since \( f_2 = s f_1 \), we have

\[
E_{2s} = s E_2
\]

where \( E_2 = 4.44 f_1 N_2 k_w_2 \Phi_m \) is the rotor emf when the rotor is standstill.

4. **T equivalent circuit**

![T equivalent circuit](image)

5. The power flow of an induction motor is illustrated in the diagram of T equivalent circuit.

\[
\text{Efficiency} = \frac{\text{Output Power}}{\text{Input Power}}
\]

6. Parameters in the T equivalent circuit can be determined by the no-load test and the locked-rotor test.

By the no-load test, the magnetisation reactance can be determined as

\[
X_m = \frac{V_{NL}}{\sqrt{3}I_{NL} \sin \phi_{NL}} \quad \text{for Y connection,}
\]

or

\[
X_m = \frac{\sqrt{3}V_{NL}}{I_{NL} \sin \phi_{NL}} \quad \text{for \( \Delta \) connection,}
\]
where $\phi_{NL} = \arccos \frac{P_{NL}}{\sqrt{3}V_{NL}I_{NL}}$.

By the locked rotor test, we can obtain

$$R_{LK} = \frac{P_{LK}}{3I_{LK}^2}$$
for Y connection,

or $$R_{LK} = \frac{P_{LK}}{3I_{LK}^2}$$ for $\Delta$ connection,

and $$Z_{LK} = \frac{V_{LK}}{\sqrt{3}I_{LK}}$$ for Y connection,

or $$Z_{LK} = \frac{\sqrt{3}V_{LK}}{I_{LK}}$$ for $\Delta$ connection.

Hence,

$$X_{LK} = \sqrt{Z_{LK}^2 - R_{LK}^2}$$

for both Y and $\Delta$ connections.

Normally, it can be assumed that

$$X_{l1} = X_{l2} = \frac{X_{LK}}{2} \quad \text{and} \quad R_{2}' = R_{LK} - R_{1}$$

where $R_{1}$ is the stator winding resistance measured immediately after the locked-rotor test.

7. The Torque/Speed, or Torque/Slip, curve can be expressed either in terms of the T equivalent circuit parameters as

$$T = \frac{3}{\omega_{syn}} \frac{V_{1e}^2R_{2}'/s}{(R_{1e}+R_{2}'/s)^2+(X_{1e}+X_{l2}')^2}$$

or in terms of the maximum torque and the corresponding slip as
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\[
T = \frac{2T_{\text{max}}}{s_{\text{max}}T} + \frac{s}{s_{\text{max}}T}
\]

where

\[
T_{\text{max}} = \frac{3}{2\omega_{\text{syn}}} \frac{V_{1e}^2}{R_{1e} + \sqrt{R_{1e}^2 + (X_{1e} + X_{l2})^2}}
\]

and

\[
s_{\text{max}}T = \frac{R_{2'}}{\sqrt{R_{1e}^2 + (X_{1e} + X_{l2})^2}}
\]

are the maximum internal torque and the corresponding slip.

8. Speed control methods:

(a) Varying number of poles to change $\omega_{\text{syn}}$ (only suitable for squirrel cage motors);

(b) Varying rotor circuit resistance by inserting external resistors into the rotor circuit (only suitable for wound rotor motors);

(c) Rotor power recovery using auxiliary devices in the rotor circuit (only suitable for wound rotor motors);

(d) Varying line voltage;

(e) Variable voltage and variable frequency (VVVF).

9. Single phase induction motors

(a) Conditions to generate a rotating magnetic field are that there exists a space displace between two phase windings, and that there exists a phase shift between two winding currents.

(b) Self-starting single phase motors are the split phase motors, the capacitor motors, and the shaded pole motors.
Exercises

1. A four pole three phase induction motor is energized from a 50 Hz supply, and is running at a load condition for which the slip is 0.03. Determine
   (a) the rotor speed in rev/min,
   (b) the rotor current frequency,
   (c) the speed of the rotor rotating magnetic field with respect to the stator frame in rev/min,
   (d) the speed of the rotor rotating magnetic field with respect to the stator rotating magnetic field in rev/min.

   Answer: 1455 rev/min, 1.5 Hz, 1500 rev/min, 0 rev/min

2. A 3 phase induction motor is wound for 4 poles and is supplied by a 50 Hz system. The stator winding is delta connected with 240 conductors/phase while the rotor winding is star connected with 48 conductors/phase. Given that the rotor winding resistance is equal to 0.013 Ω/phase, the rotor leakage reactance equal to 0.048 Ω/phase when the rotor speed \( \omega_r=0 \), the supply voltage equal to 400 V, \( K_d=0.96 \) and \( K_p=1.0 \) for each winding, and the impedance of the stator winding is negligible, calculate
   (a) the flux per pole,
   (b) the rotor emf per phase at standstill with rotor windings open circuited,
   (c) the rotor emf and current per phase at a slip of 0.04,
   (d) the phase difference between the rotor emf and current at a slip of 0.04.

   Answer: 0.01565 Wb/pole, 80 V, 243.5 A, 8.45°

3. The parameters of the equivalent circuit for a 3 phase 4 pole star connected 50 Hz induction motor are
   \( R_1=0.2 \) Ω, \( R_2=0.1 \) Ω, \( X_1=0.5 \) Ω, \( X_2'=0.2 \) Ω, \( X_m=20.0 \) Ω
   The supply voltage is 220 V and the total iron and mechanical losses are 350 W. For a slip of 2.5%, calculate
   (a) the stator current,
   (b) the output power,
   (c) the output torque, and
   (d) the efficiency.

   Answer: 30 A, 9.58 kW, 62.55 Nm, 89.1%

4. A 3 phase star connected 220 V (line to line) 7.37 kW 60 Hz 6 pole induction motor has the following constants in ohms per phase referred to the stator:
   \( R_1=0.294, \quad R_2'=0.144, \quad X_1=0.503, \quad X_2'=0.209, \) and \( X_m=13.25 \)
The total friction, windage, and core losses may be assumed to be constant at 403 W, independent of load. Neglect the impedance of the power supply. For a slip of 2.0% and when the motor is operated at the rated voltage and frequency, calculate (a) the speed, (b) the output torque and power, (c) the stator current, (d) the power factor, and (e) the efficiency.

*Answer: 1176 rev/min, 42.5 Nm, 5230 W, 18.8 A, 0.844 lagging, 86.3%*

5. For the motor in question 4 at a slip of 0.03, determine (a) the load component $I_2'$ of the stator current, (b) the internal torque, i.e. electromagnetic torque, and (c) the internal power.

*Answer: 23.9 A, 65.5 Nm, 7.97 kW*

6. For the motor in question 4, calculate (a) the maximum internal torque and the corresponding speed, and (b) the internal starting torque and the corresponding stator current.

*Answer: 175 Nm, 970 rev/min, 78.0 Nm, 150.5 A*