TO BE RETURNED AT THE END OF THE EXAMINATION.
THIS PAPER MUST NOT BE REMOVED FROM THE EXAM CENTRE.

SURNAME: _______________________
FIRST NAME: _______________________
STUDENT NUMBER: _______________________
COURSE: _______________________

SPRING SEMESTER, 2004

SUBJECT NAME : ELECTRICAL ENERGY TECHNOLOGY
SUBJECT NO. : 48550
DAY/DATE : TUESDAY 23 NOVEMBER 2004
TIME ALLOWED : 3 Hours plus 10 Min. reading time
START/END TIME : From 18:00 to 21:10

NOTES/INSTRUCTIONS TO CANDIDATES:

- ONLY NON-PROGRAMMABLE CALCULATORS MAY BE USED.
- ONE HANDWRITTEN DOUBLE SIDED A4 SHEET WITH WHATEVER THE STUDENT WISHES TO WRITE ON IT MAY BE TAKEN INTO THE EXAM ROOM.
- ROUGH WORK CAN BE DONE ON THE LAST THREE BLANK PAGES AT THE END OF EACH ANSWER BOOK

Examiner: Prof. J.G. Zhu
Assessor: Dr. G. Boardman
Problem One
List five (5) types of available energy resources and briefly describe the methods to convert them into electricity. Indicate if they are clean and/or sustainable.

Problem Two
A super capacitor is rated as 1000 F 30 V DC.
(a) Calculate the maximum energy this super capacitor can store.
(b) If this super capacitor is fully charged and employed to provide energy to a resistive load \( R_L = 50 \, \Omega \) as shown in Fig.P2 below, calculate the amount of energy the load receives in the first second.

Problem Three
For the following assumptions:
- Cost of a 3 phase high voltage transformer is $100 per kVA;
- Cost of formed aluminium conductor is $6 per kg;
- Cost of each tower is $(5,000+84V+10m)$ where \( V \) is the rated line-to-line voltage in kV, and \( m \) is total mass of conductor between towers in kg;
- Distance between towers is 100 m;
- Mass density of aluminium is 2700 kg/m³; and
- Current density in the conductor is 0.7 A/mm².

Find the optimum voltage for a 300 km 3 phase transmission line with aluminium conductors to transmit a continuous power of 500 MW at power factor 0.8 lagging.

Problem Four
A 3 phase, 2 pole, delta connected, 22 kV (line to line), 300 MVA round rotor synchronous machine is operated as a 50 Hz generator. With a rotor current of 500 A, the open circuit generated voltage is 22 kV (line to line) at 50 Hz, and the short circuit current is 10,000 A. Assuming a linear open circuit characteristic and negligible stator winding resistance,
(a) Determine the synchronous reactance at 50 Hz;
(b) Calculate the required rotor current for operation at rated stator current and voltage with 0.8 lagging power factor;
(c) Calculate the load angle for (b);
(d) Calculate the voltage regulation for (b); and
(e) Show that the output power can be expressed as

\[
P_{out} = \frac{3E_1V_1}{X_s} \sin \delta
\]

where \( E_1 \) is the stator emf per phase, \( V_1 \) the stator voltage per phase, \( X_s \) the synchronous reactance, and \( \delta \) the load angle.

(20 marks)
Problem Five
A 3-phase, 4-pole, star connected, 415 V (line to line), 50 Hz, induction motor has the following T equivalent circuit parameters:
\[ R_1 = 0.75 \, \Omega, \quad R_2' = 0.80 \, \Omega, \quad X_{l1} = X_{l2}' = 2.15 \, \Omega, \quad X_m = 50.36 \, \Omega, \quad \text{and} \quad R_c \text{ can be ignored.} \]
The total friction, windage, and core losses may be assumed to be constant at 700 W, independent of load.

(a) Derive an expression for the electromagnetic torque developed by the motor for a given stator supply voltage, in terms of the slip.

(b) Calculate the starting electromagnetic torque and the corresponding stator current.

(c) Calculate the maximum electromagnetic torque and the corresponding rotor speed.

(d) Sketch the torque/speed curve for 415 V (line to line), 50 Hz operation, using the results of (a) and (b).

(e) Calculate the output torque and the output power at slip 0.025.

(20 marks)

Problem Six
Fig.P6 shows the cross section of a switched reluctance motor. The rotor is in such a position that the overlap angle between poles aa' of the rotor and poles AA' of the stator is 15°. The air gap \( g = 0.5 \, \text{mm}, \) stator bore radius \( r = 25 \, \text{mm}, \) and axial length \( l = 50 \, \text{mm}. \) There are 98 turns on each stator pole.

(a) A current of 6 A flows through the two coils A and A' in series. Sketch the flux paths on Fig.P6 for this condition.

(b) Calculate the flux density in the air gap between the poles a and A for the conditions in part (a).

(c) For the conditions in part (a), calculate the torque and indicate its direction. Neglect fringing and leakage, and assume that the steel parts are infinitely permeable.

(d) Through what angle of rotation is the torque essentially constant, if the current is constant and there is no fringing?

(20 marks)
Problem Seven
A 230 V, DC shunt motor has an armature circuit resistance of 0.25 Ω and a field circuit resistance of 115 Ω. At full load the motor draws a current of 40 A from the power supply and the speed is measured at 1050 rev/min. Neglect saturation (assuming linear magnetisation curve).
(a) Find the electromagnetic torque in Nm at full load;
(b) The field rheostat is adjusted so that the resulting field circuit resistance is 144 Ω. Find the new operating speed in rev/min assuming the electromagnetic torque is kept the same value as in part (a); and
(c) Assuming that rotational losses due to friction and windage amount to 600 W, calculate the efficiency in part (b).

(10 marks)
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Problem 1. Solution:

1) Fossil fuels, e.g. coal and oil
   - Coal → steam → turbine → Generator → Electricity
   - Diesel → petrol → Engine → Generator → Electricity
   * CO₂ emission (Not clean), and expandable.

2) Hydro-electric
   - Water + dam → Turbine → Generator → Electricity
   * Clean and renewable.

3) Wind energy
   - Wind → Turbine → Generator → Electricity
   * Clean and renewable.

4) Solar energy
   - Solar cells → Electricity
   - Solar heat collector → Steam → Turbine → Generator
   * Clean and renewable

5) Nuclear energy
   - Nuclear reactor → Steam → Turbine → Generator → Electricity
   * Can be clean, but the nuclear waste disposal is difficult. Expandable.

6) Geothermal energy
   - Geothermal collector → Steam → Turbine → Generator → Electricity
Problem 1. Solution (continued).

* Clean but expandable in nature.

(7) Bio-mass

Bio-mass → Steam → Turbine → Generator → Electricity

* CO₂ emission (not clean), but renewable.

(8) Wave and tide

Wave and tide (+ dam) → Turbine → Generator → Electricity

* Clean and renewable.
Problem 2. Solution:

(a) \( W = \frac{1}{2} C V_c^2 = \frac{1}{2} \times 1000 \times 30^2 = 4.5 \times 10^5 \) (J)

(b) \( V_c = I_c R_L \) and \( I_c = C \frac{dV_c}{dt} \)

\[ \therefore \frac{-1}{R_L C} \frac{dt}{V_c} = \frac{1}{V_c} \frac{dV_c}{dt} \]

Hence, \( \ln V_c = -\frac{1}{R_L C} t + A_t \)

where \( A_t \) is a constant to be determined by the initial condition.

or \( V_c = A_t e^{-\frac{t}{R_L C}} \) where \( A_t = e^{A_t} \).

When \( t = 0 \), \( V_c = 30 \) (V), and therefore

\[ A_t = \left. V_c \right|_{t=0} = 30 \) (V)

\[ I_c = \frac{V_c}{R_L} = \frac{A_t}{R_L} e^{-\frac{t}{R_L C}} \]

\[ P_L = I_c^2 R_L = \left( \frac{A_t}{R_L} \right)^2 e^{-\frac{2t}{R_L C}} \]

\[ \therefore W_L = \int_0^t P_L dt = \frac{A_t^2}{R_L} \int_0^t e^{-\frac{2t}{R_L C}} dt \]

\[ = -\frac{C}{2} \frac{A_t^2}{R_L} e^{-\frac{2t}{R_L C}} \bigg|_0^t = CA_t^2 \left( 1 - e^{-\frac{t}{R_L C}} \right) \]

\[ = 18 \) (J) \]
Problem 3. Solution:

\[ S = \frac{500 \times 10^3 \text{ (kW)}}{0.8} = 6.25 \times 10^5 \text{ (kVA)} \]

Cost of Transformer = $100 \times 6.25 \times 10^5 \times 2

+ $1,25 \times 10^6

Cost of conductors = $6 \times \frac{6.25 \times 10^5 \times 10^3}{\sqrt{3} \times 10^3 \times 0.7 \times 10^6}

+ $13 \times 2700 \times 3 = \frac{7515.8633 \times 10^6}{V}

Cost of towers = \$(5000 + 84V + 10 \times 6.25 \times 10^5 \times 100 \times \frac{x^2}{1000000} \times 2700 \times 3 \times \left( \frac{300 \times 10^3}{100} + 1 \right) \times \frac{1}{V})

\text{Total Cost}

Let \( \frac{\text{dTotal Cost}}{dV} = 0 \)

\[ 0 = -\frac{7515.8633 \times 10^6}{V^2} + \frac{84}{V^2} - \frac{4.1754 \times 96 \times 10^6}{V^2} \times 3001 = 0 \]

and we have

\[ V = \sqrt{\frac{7515.8633 \times 10^6 + 4.1754 \times 96 \times 10^6 \times 3001}{84 \times 3001}} \]

\[ = 281.9982 \text{ (AV)} \approx 282 \text{ (kV)} \]

\[ \checkmark \]
Problem 4. Solution:

(a) \( X_s = \frac{V_o}{I_{sc}} = \frac{22 \times 10^3}{10^2 / \sqrt{3}} = 3.81 (\Omega) \)

(b) \( I_a = \frac{S}{3 V_a} = \frac{300 \times 10^6}{3 \times 22 \times 10^3} = 4.65 \times 10^4 \) (A)

\[ E_a = V_a + jX_s I_a \]

\[ = 22 \times 10^3 + j(3.81k \cdot \frac{10^4}{22} \cdot \text{angle} - 36.87°) \]

\[ = 22 \times 10^3 + j1.75 \times (0.8 - j0.6) = (32.3923 + j13.856k) \times 10^3 \]

\[ = 35.2315 \times 10^3 \text{angle} 23.16° \text{ (V)} \]

\( \therefore I_f = \frac{35.2315 \times 10^3}{22 \times 10^3} \times 500 = 800.72 \text{ (A)} \)

(c) \( \delta = 23.16° \)

(d) \( VR = \frac{V_{NL} - V_{Rated}}{\sin \delta} = \frac{E_a - V_{Rated}}{V_{Rated}} = \frac{35.2315 - 22}{22} \)

\[ = 60.14\% \]

(e) \( P_{out} = 3V_a I_a \cos \phi \)

From the phasor diagram, we have

\[ \frac{E_a}{\sin(90° + \phi)} = \frac{X_s I_a}{\sin \delta} \text{ or } I_a \cos \phi = \frac{E_a \sin \delta}{X_s} \]

\[ \therefore P_{out} = \frac{3E_a V_a \sin \delta}{X_s} \]
Problem 5. Solution:

(a) \[ I_1 = \frac{V_1}{R_1} \quad I_2 = \frac{V_1}{\sqrt{\left(R_{le} + \frac{R}{s}\right)^2 + \left(X_{le} + X_n\right)^2}} \]

The power dissipated in \( \frac{1}{s}R_2 \) equals the power converted into the mechanical power \( T \).

Thus,

\[ T = \frac{3 \cdot I_2 I_2^* \frac{1}{s} R_2}{\omega_p} \]

Where \( V_{le} = \frac{X_{le} V_1}{\sqrt{R_{le}^2 + (X_{le} + X_n)^2}} \), \( \omega_p = \text{Real} \left( \frac{R + jX_e}{jX_n} \right) \), and 

\[ \omega_{syn} = \frac{2\pi f_1}{p/2} \]

\[ \omega_{syn} = \frac{2\pi f_1}{p/2} \]

\[ \therefore T = \frac{3}{\omega_{syn}} \cdot \frac{V_{le}^2 \cdot \frac{R}{s}}{\left( R_{le} + \frac{R}{s} \right)^2 + \left( X_{le} + X_n \right)^2} \]

\[ \approx \frac{3}{f_{0,\text{on}}} \cdot \frac{(4.45)^2 \cdot 0.8}{s} \left( 0.75 + \frac{0.8}{s} \right)^2 + (2.4 + 2.4)^2 \]
Problem 5. Solution. (continued)

\[ T = \frac{877.135 \, / \, s}{(0.75 + \frac{0.8}{s})^2 + 18.49} \]

(b) \[ T_{sf} \bigg|_{s=1} = \frac{877.135}{(0.75 + 0.8)^2 + 18.49} = 41.983 \, (Nm) \]

\[ I_{sf} = \frac{V_i}{n(\sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2})} \]

= \frac{415}{\sqrt{n^2}} = 52.419 \, (A) \]

(c) \[ T_{max} = \frac{3}{2 \times 50 \times \pi} \cdot \frac{V_{ie}^2}{R_{ie} + \sqrt{R_{ie}^2 + (X_{ie} + X'_{ie})^2}} \]

\[ \approx \frac{3}{2 \times 50 \times \pi} \cdot \frac{(415/\sqrt{13})^2}{0.75 + \sqrt{0.75^2 + (2.15 + 2.15)^2}} \]

= 107.179 \, (Nm) \]

\[ S_{T_{max}} = \sqrt{R_{ie}^2 + (X_{ie} + X'_{ie})^2} \approx \frac{0.8}{\sqrt{0.75^2 + (2.15 + 2.15)^2}} \]

= 0.1833 \, or \, 1028.2902 \, (rad/s) \]

or \, 1225.08 \, (rev/min) \]
Problem 5. Solution (continued)

(d) \[ T = \frac{877.135}{0.02} \left( 0.75 + \frac{0.08}{0.025} \right)^2 + 18.49 \]
\[ = 32.457 \text{ (Nm)} \]

\[ CW_r = (1 - S) \cdot CW_{syn} = (1 - 0.025) \times 5.0 \times 11 = 53.53 \text{ (rpm)} \]

\[ T_{loss} = \frac{700}{153.153} = 4.571 \text{ (Nm)} \]

\[ T_{out} = T - T_{loss} = 32.457 - 4.571 = 27.886 \text{ (Nm)} \]

and \[ P_{out} = T_{out} \cdot CW_r = 4.024.266 \text{ (W)} \]
Prob. 6. Solution:

(a)

(b) \( N_1 = 98 \times 6 = 588 \) (A-turn)

\[
R_g = \frac{\mu_0 A_g}{\mu_0 0.01 \times 0.01} = \frac{0.5 \times 10^{-3}}{4\pi \times 10^{-7} \times 15^\circ \times \frac{\pi}{180} \times 25 \times 10^{-3} \times 50 \times 10^{-3}}
\]

\[= 121.5854 \text{ } \Omega \]

\[
\phi = \frac{2N_1}{2R_g} = \frac{2 \times 588}{2 \times 121.5854 \text{ } \Omega}
\]

\[= 4.8361 \times 10^{-3} \text{ } \Phi \]

\[
\beta_g = \frac{\phi_1}{A_g} = \frac{4.8361 \times 10^{-3}}{15^\circ \times \frac{\pi}{180} \times 25 \times 10^{-3} \times 50 \times 10^{-3}}
\]

\[= 1.44 \text{ } \text{T} \]

or: By Ampere’s law, \( 2A_g \times g = 2N_2 \)
\[ B_y = M_0 \frac{H_y}{g} = M_0 \frac{k}{g} = \frac{4\pi \times 10^{-7} \times \frac{5 \times 10^{-3}}{0.5 \times 10^{-3}}}{g} = 1.48 \quad (T) \]

\[ L = \frac{(2N)^2}{2R_y} = \frac{2N^2}{2 \times \frac{J}{u \cdot \omega \cdot r}} = \frac{2N^2}{u \cdot \omega \cdot r} \]

\[ W_f' (2, \omega) = \frac{1}{2} L \omega^2 = \frac{N^2 \omega^2 \cdot u \cdot \omega \cdot r}{J} \]

\[ T = \frac{\Theta W_f' (2, \omega)}{2} = \frac{1}{2} \frac{dL}{d\Theta} = \frac{N^2 \omega^2 \cdot u \cdot \omega \cdot r}{J} \]

\[ = \frac{580^2 \times 4 \pi \times 10^{-7} \times 35 \times 10^{-3} \times 50 \times 10^{-3}}{0.5 \times 10^{-3}} \]

\[ = 1.086 \quad (Nm) \quad \text{(Clockwise)} \]

(d) Assume \( \Theta = 0^\circ \) when \( a \) and \( A \) are aligned. Then torque \( T \) is constant when \( 0^\circ < \Theta < 30^\circ \).
Problem 7. Solution:

(a) \[ I_f = \frac{V_f}{R_f} = \frac{230}{115} = 2 \text{ (A)} \]

\[ I_a = I_f - I_f = 40 - 2 = 38 \text{ (A)} \]

\[ E_a = V_a - R_a I_a = 230 - 38 \times 0.25 = 220.5 \text{ (V)} \]

\[ K_a \Phi = \frac{E_a}{I_f} = \frac{220.5}{1050 \times \frac{2\pi}{60}} = 2.0054 \text{ (V.s/rad)} \]

\[ \therefore T = K_a \Phi I_a = 2.0054 \times 38 = 76.20 \text{ (Nm)} \]

(b) \[ I_f = \frac{V_f}{R_f} = \frac{230}{144} = 1.5972 \text{ (A)} \]

Assuming linear magnetisation curve, we have

\[ K_a \Phi' = \frac{I_f}{I_f} \times K_a \Phi = \frac{1.5972}{2} \times 2.0054 \]

\[ = 1.6015 \text{ (V.s/rad)} \]

If the electromagnetic torque is kept the same, the new armature current is

\[ I_a = \frac{T}{K_a \Phi'} = \frac{76.20}{1.6015} = 47.5794 \text{ (A)} \]

\[ E_a' = V_a - R_a I_a = 230 - 0.25 \times 47.5794 = 218.1052 \text{ (V)} \]

\[ \text{or} \quad \omega_r = \frac{E_a'}{K_a \Phi'} = \frac{218.1052}{1.6015} = 136.19 \text{ (rad/s)} \]

\[ = 1300.50 \text{ (rev/min)} \]
(c) \[ P_{out} = T \cdot \cos \theta - P_{net} = 76.20 \times 136.19 - 600 = 9777.67 \text{ W} \]

\[ P_{in} = V \cdot I = 230 \times (47.5794 + 1.5972) = 11310.61 \text{ W} \]

\[ \eta = \frac{P_{out}}{P_{in}} = \frac{9777.67}{11310.61} = 0.8645 = 86.45\% \]